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Geometric Design in Islamic Architecture: Examination of Tessellation Configurations in Mosques

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Abstract

Tessellation is a mathematical concept used in geometric design. As in many fields, this method is also encountered in architecture. In this study, it is aimed to investigate the tessellation configurations in marble and wooden-columned mosques within the scope of geometric design in architecture, to show the effectiveness of geometry in the architectural production in these buildings, and to draw attention to the relationship between tessellation and geometry in Islamic buildings. The research question of the study is how tessellation configurations are applied in marble and wooden-columned mosques in Islamic architecture and how this relationship is established. Depending on this research question, Beyşehir Eşrefoğlu Mosque, Sivrihisar Ulu Mosque, and Konya Alâeddin Mosque in Turkey were examined within the scope of this study. The common point of these Islamic buildings is the use of tessellation patterns in the minbar and mihrab sections. Content analysis was performed by looking at the architectural features of the marble and wooden-columned mosques and geometric analysis of the tessellation configurations in the minbars and mihrabs was conducted with the data obtained. As a result, it has been seen that various variations in tessellation configurations are used in marble and wooden-columned mosques, which are products of Islamic architecture. The academic study in which the tessellation method is emphasized in mosques is limited. Therefore, this study will fill the gap in the literature.

Keywords: Geometric design, Geometric shape, Minbar, Pattern, Tessellation, Islamic architecture, Wooden-columned mosque

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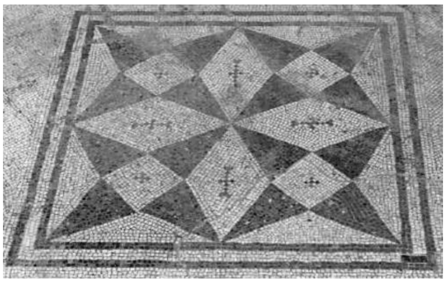
Introduction

Architecture is a multidisciplinary, bridging art and science together and that is fed by various disciplines. Some of these disciplines are geometry and mathematics. Architectural designs are created based on the rules of geometry and mathematics (Takva et al., 2023). Tessellations also contribute to the architectural production process as a method of geometry and mathematics. This method consists of geometric shapes. The concept of tessellation, which allows two or three-dimensional designs, is a set of patterns that are shaped as a juxtaposition of geometric figures without overlapping or leaving any spaces between them (Lang, 2017). The use of tessellation is common, especially in arts and engineering (Paulino and Gain, 2015). The concept of tessellation has a long history and dates back to before the Sumerians. It was used to create decoration material on the walls of the buildings of the Sumerians (Cheng et al., 2018). There are traces of this geometric method in many civilizations. It appears in the architecture of Ancient Egypt, Byzantium, Moors, Romans, Greece, Persia, Arabia, China, and Japan (Fig. 1). Tessellations are found in floor and ceiling ornaments, Persian ornaments, and Byzantine motifs in ancient Romans (Fatta and Mediati, 2020; Chang, 2018). Geometric patterns are seen on the floors of the Basilica of Santa Maria in Rome, Italy, and the Villa Romana del Casale in Sicily (Wichmann and Wade, 2017; URL-1). There are also geometric mosaics on the walls and doors of the Tomb of Itimad-ud-Daulah (Sharma and Gupta, 2017).

(a)

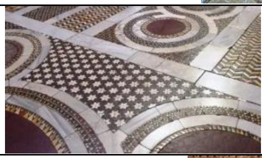
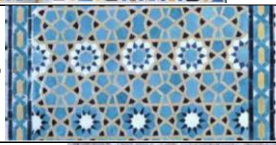
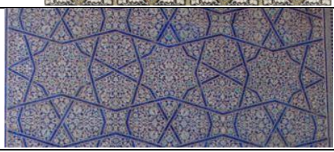


(b)

Fig. 1: (a) Wall decorations used in the Uruk and Jemdet Nasr periods of Southern Mesopotamia (URL-2) and (b) Roman tiling in Pompeii, Italy (Fathauer, 2020)

Tessellation patterns are also found in Islamic architecture. It is seen that tessellations are formed with a star pattern on the ceramics on the walls of Arg of Karim Khan Zand in Shiraz, the Friday Mosque (Masjid-i-Jami) in Kerman, and Imamzadeh Darb-e Imam in Isfahan (Bonner, 2017; Sarhangi, 2012). There are on the facades of Tosh Hovli Palace in Uzbekistan (Wichmann and Wade, 2017), on the wall decorations of the Mama Hatun Mausoleum in Tercan, Turkey (Lu and Steinhardt, 2007), on the Ahlat tombstones (Agirbas, 2020), on the walls of the Great Mosque of Kairouan (Abdullahi and Embi, 2013), the tomb towers of Kharraqan (Bier, 2012), the Humayun Tomb in Delhi (Cromwell, 2010), and the walls of the Alhambra Palace (Lim, 2022) tessellation configurations. In Table 1, the patterns of architectural buildings in which tessellations are used are given chronologically. This geometric method, observed in ancient and Islamic architecture, has also been the focus of the attention of mathematicians and artists. Maurits Cornelis Escher (1898–1972), is an artist who works with mathematical concepts, produced works on the basis of mathematics and art (Ouyang et al., 2022). He created tessellation patterns using animals and natural objects. It is thought that the formation of these patterns was indirectly influenced by Islamic patterns, as in the Alhambra Palace (Eberle, 2014). Alain Nicolas is also a tessellation artist and has proposed 35 isohedral tiles similar to Escher's tessellation patterns (Huang et al., 2022).

Table 1: Chronology of architectural buildings in which tessellation configurations are applied

Construction century of the building	Name of the building and tessellation configuration
4th	 Villa Romana del Casale
9th	 The Great Mosque of Kairouan
11th	 Basilica of Santa Maria  Tomb towers of Kharraqan
12th	 Ahlat tombstones
13th	 Alhambra Palace  Mama Hatun Mausoleum
14th	 Friday Mosque (Masjid-i-Jami)
15th	 Imamzadeh Darb-e Imam
16th	 Humayun Tomb
17th	 The Tomb of Itimad-ud-Daulah
18th	 Arg of Karim Khan Zand
19th	 Tosh Hovli Palace

In addition to the tessellation configurations of mathematicians and artists from the past to the present, there are literature studies that will contribute to future studies. Abdullahi and Embi (2013) conducted detailed research on the decorative patterns of 100 buildings in the world of Islamic architecture. Patterns were analyzed and arranged chronologically. Focusing on the origins and artistic values of Islamic geometric patterns, the evolution of patterns investigated until the end of the 18th century is explained. Datta et al. (2014) developed a facade system consisting of tessellation patterns to provide advantages in features such as sun shading, light control, and thermal gain on building facades. It has adopted a dynamic approach by making an experimental simulation. After the performance analysis according to multiple criteria in the pattern in which pentagon geometric shape is used, the dynamic movements of the facade are reviewed. Khouri (2017) used Islamic geometric patterns on gridshell structures. It has been shown that the geometry in the structures gives precision to the model and aimed to investigate the contributions of the pattern topology to the structure. He observed that the shell structure with the highest geometric density gave the best structural analysis performance. Gadaleta (2018) produced a cut stone dome with a semi-regular tessellation pattern after making a historical analysis of hemispherical domes. It has been shown that the dome structure can be optimized compared with conventional methods. Ross et al. (2020) developed a tessellation wall system. Physical and analytical structural analyses were carried out on this building element, which is formed because of repetitive reproduction of a geometric shape. Because of the analysis, it has been observed that regional cracks occur compared with a normal reinforced concrete wall. Rodriguez-Padilla et al. (2021) discussed how to produce a structure with a tessellation pattern on a non-planar surface with a 3D printer. A mathematical algorithm was developed and trials were carried out within the framework of the printing trajectory. As a result, it was found that the resolution of the tessellated surface is central to the error.

Based on the information obtained from the literature, it has been seen that tessellation configurations have a significant place in Islamic architecture. The fact that a regular geometric form is not used in the buildings emphasizes the unlimited use of geometry. It is observed that geometric studies have increased in architectural construction approaches with the mathematical calculations performed by geometric methods. It can be said that tessellation, which is one of the geometric design method in which different geometries are used alone or can be integrated with more than one geometric shape, is used to ensure this situation. In this study, it is aimed to investigate tessellation configurations in Beyşehir Eşrefoğlu Mosque, Sivrihisar Ulu Mosque, and Konya Alâeddin Mosque within the scope of geometrical design in Islamic architecture. After a comprehensive literature review, the tessellation configurations used in the minbars and mihrabs of marble and wooden-columned mosques were analyzed. Content and geometric analyses were performed.

Material and method

The research question of the study is how tessellation configurations, which play an active role in arts and geometry, are applied in Islamic architecture and how this relationship is established. Based on this research question, mosques in Turkey, which were designed according to the principles of tessellation, were examined within the scope of the study. The aim of this study is to examine the buildings of Islamic architecture in which tessellation configurations are applied and to show the effectiveness of geometry in the architectural production. Additionally, depending on the characteristics of the buildings, it was desired to create a reference to prepare the ground for different geometric formations in future studies. First, content analyses were conducted by looking at the architectural features of these buildings. The geometric analysis framework of the buildings was created with the data obtained after the content analyses. It is aimed to highlighting the relationship between tessellation and marble and wooden-columned mosques. To this result, minbars and mihrabs in marble and wooden-columned mosques, which are Islamic architectural building products, in which different tessellation configurations are applied, were selected and evaluated. Tessellation modules and formation patterns of the minbar and mihrab geometries are shown. The flow chart of the study is given in Fig. 2.

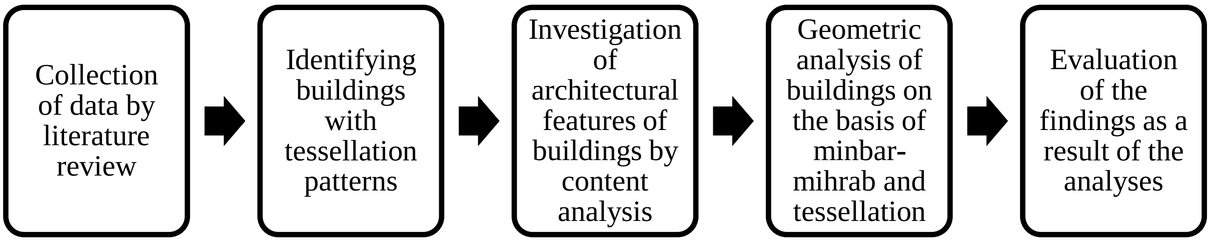


Fig. 2: Step-by-step demonstration and flowchart of the study

Tessellation can be defined as a collection of shapes in a mathematical framework covering any surface. To apply the concept of tessellation and its configurations, an understanding of geometric shapes and their combinations is required. The edge and angle connections of geometric shapes are central to the creation of tessellation configurations. There are certain rules for the implementation of these configurations. In addition to placing the geometric shapes in a sequential order, without spaces and without overlapping, it should be ensured that modules that complete 360 degrees are obtained (Fig. 3). For this reason, a mathematical sequence must be created. The mathematical expression of tessellation formed by regular polygons includes writing the number of sides of each polygon sequentially in clockwise order around the intersecting midpoint of a module (vertex of the tessellation) (Gazi and Korkmaz, 2015). Surfaces are obtained by multiplying a tessellation module that meets the mathematical conditions. The duplication process is performed using the repetition principle, which is a basic principle of architectural design. Surfaces created with tessellation modules can be designed in 2 and 3 dimensions.

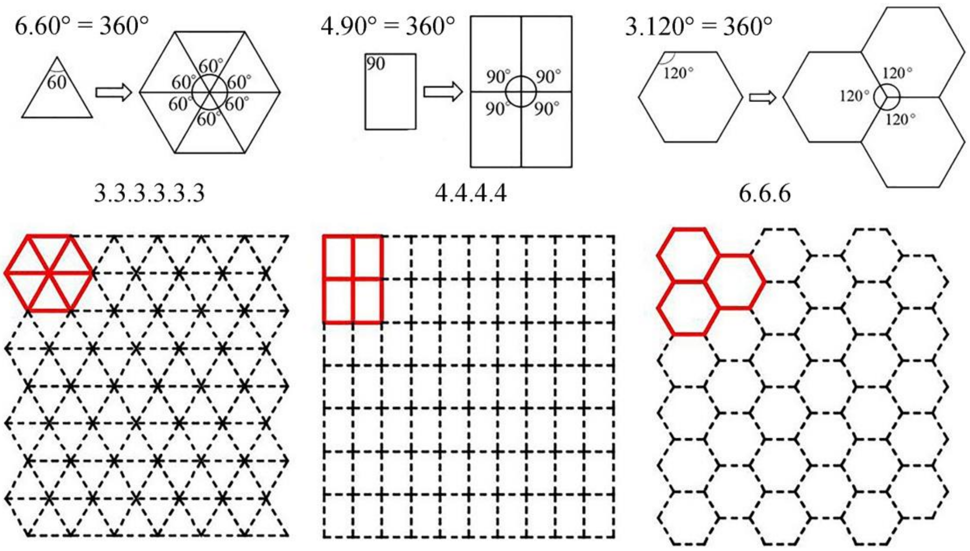


Fig. 3: Mathematical representation of tessellation modules and creation of tessellation configurations

The number of sides of geometric shapes completing 360 degrees in a tessellation module is important in the mathematical expression (Friedenberg, 2019). This expression is created by the number of sides of the geometric shapes in a tessellation module and how many geometric shapes are used (Ross et al., 2020). With 6 of the equilateral triangles coming together to complete 360 degrees, 4 of the rectangular coming together, and 3 of the regular hexagons coming together, the mathematical sequences in Fig. 3 occurs. While regular tessellations are formed with tessellation configurations formed by a regular polygon, tessellation configurations formed by integrating more than one regular polygon are called semi-regular tessellations. Tessellations occur because of translation, reflection, and rotation movements of a geometric form. Various integrations can be obtained with translation, reflection, and rotation movements (Fig. 4). The definitions of translation movement of a geometric shape from one place to another without changing its dimensions, reflection to its symmetry with respect to a certain angle or line, and rotation movement to change direction while preserving its size according to a certain degree are made (Değer and Değer, 2012). In the production of tessellation patterns, various configurations can be obtained with one of these variations or their multiple combinations (Ždímalová, 2020).

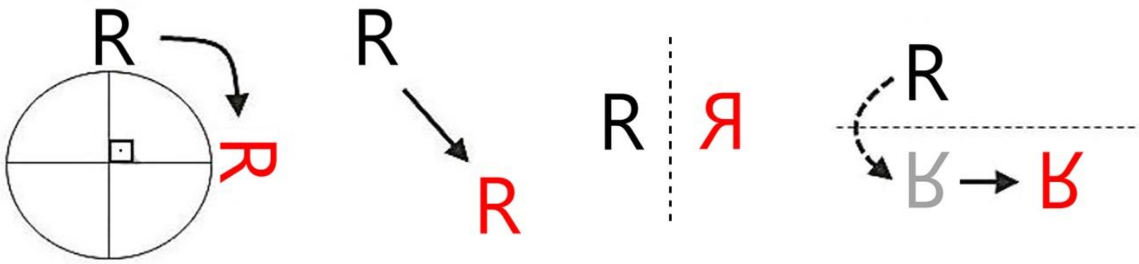
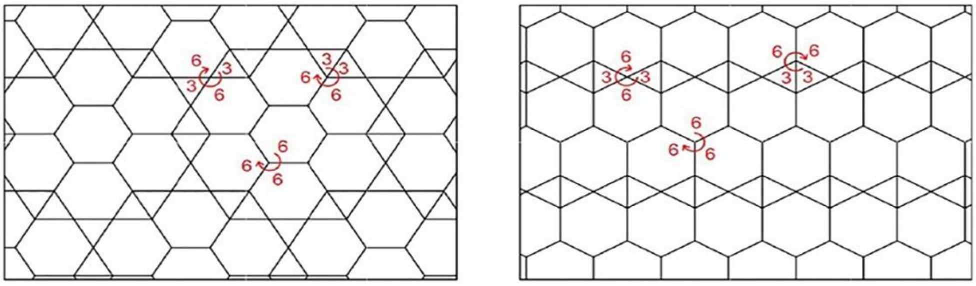
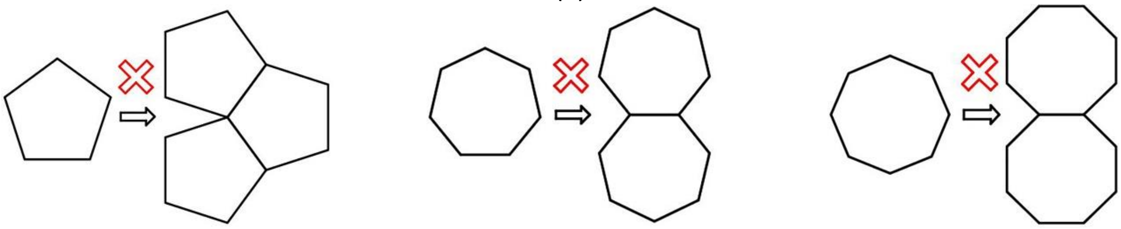

(a) (b) (c) (d)

Fig. 4: The movements that make up the tessellations; (a) rotation, (b) translation, (c) reflection, and (d) combinations of these movements

In addition to the application of the tessellation method with regular geometric shapes, tessellation configurations can be created with irregular free-form geometric shapes. While more stable and rigid configurations are obtained with regular tessellation, irregular free-form tessellations create a more flexible looking and variable pattern class (Sotelo Calvillo, 2020). Semi-regular tessellation patterns can also be created with rotation, translation, reflection, and combinations of these movements. Semi-regular tessellations in which at least two different geometric shapes can form patterns show that complex geometric combinations can be created (Fig. 5a). In a tessellation configuration, the greater the diversity of geometric shapes, the greater the diversity of mathematical expressions. Not all regular polygons are suitable for tessellation patterns. Tessellation configurations do not occur when geometries such as regular pentagon, heptagon, octagon, nonagon, and decagon, except triangle, rectangular and hexagonal geometries, are brought together due to angle and edge connections (Fig. 5b). This is the case when the same geometric shapes are brought together. Tessellation configurations can be created when integrated with different geometric shapes. Tessellation patterns were based on mathematical and geometric information. Some geometric shapes alone cannot form patterns, but they can produce semi-regular or free-form tessellations with a polygon with a different number of sides completing 360 degrees (Akiyama et al., 2010).



(a)



(b)

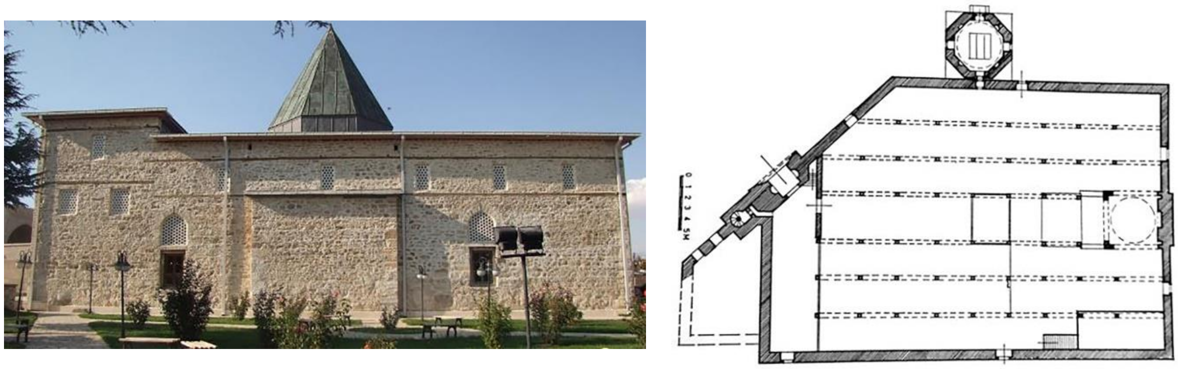
Fig. 5: (a) Representation of mathematical sequences created according to the number of sides of geometric shapes in semi-regular tessellation patterns (Weizmann et al., 2017), (b) regular pentagon, heptagon, and octagon geometric shapes in which the tessellation pattern is broken

Case studies

Geometric decisions and shapes, which are central to the identity of the building, bring freedom and originality to the buildings in cooperation with the architectural characteristics. Many application areas in which tessellation configurations are applied in areas close to architecture are found in the literature. In the field of architectural studies, there are various systems in which this understanding of geometric order is applied. One of these systems is the tessellation patterns seen in building elements. The geometrical order relationships seen in the building elements (wall and ceiling decorations and floor coverings) show the originality of the building. Geometrical order relations contribute to the originality in marble and wooden-columned mosques. Regular and semi-regular tessellation patterns are encountered in marble and wooden-columned mosques. When the marble and wooden-columned mosques in Turkey are examined, tessellation configurations are seen in the minbar and mihrab sections as a common feature in these Islamic buildings. In this section, examples of minbars and mihrabs with different tessellation configurations were examined. In the examined Islamic buildings, geometric shape, tessellation module, tessellation module formation, mathematical sequence parameters are focused.

Beyşehir Eşrefoğlu Mosque

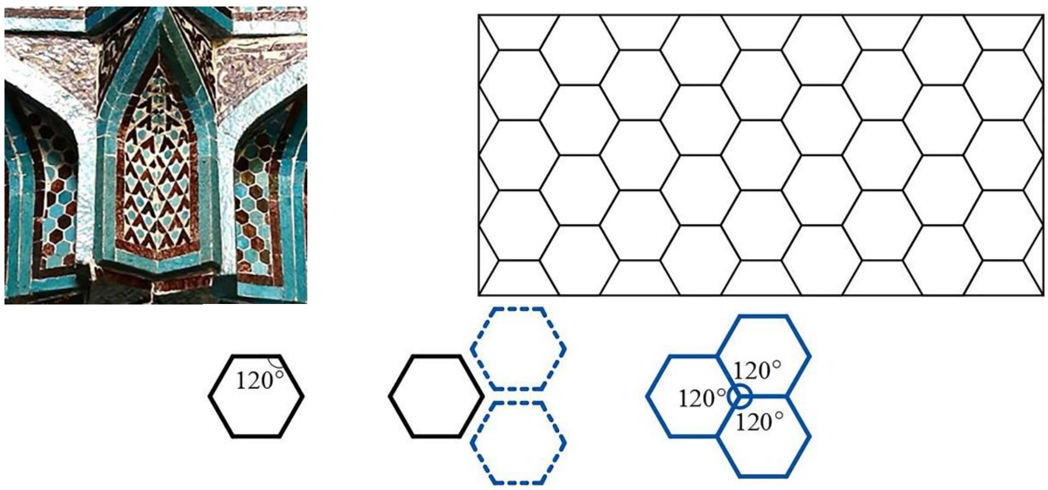
Beyşehir Eşrefoğlu Mosque, one of the wooden-columned mosques built during the Anatolian Seljuk Period, has an important and valuable place in Anatolian Turkish-Islamic architectural stock. The mosque, which was included in the UNESCO's Provisional World Heritage Sites list in 2012, is one of the largest and original examples of the wooden-columned flat ceiling mosque tradition in Anatolia. It was built by Süleyman Bey, the founder of the Eşrefoğlu Principality, between 1296–1299 in Beyşehir, Konya. In addition to the entrance facade of the mosque, there are entrances from the east and west of the building. In the built environment where the mosque inside the complex is located, there are tombs, khan (caravanserai), a Turkish bath, and a public fountain outside the mosque. The minaret of the mosque, which was built with a masonry construction system, consists of a brick body rising on a cut stone pedestal. The building, which has external dimensions of 31.77*46.55 meters, is the largest of Anatolia's wooden-columned mosques (Erarslan, 2021). The external view and plan of the mosque are given in Fig. 6.



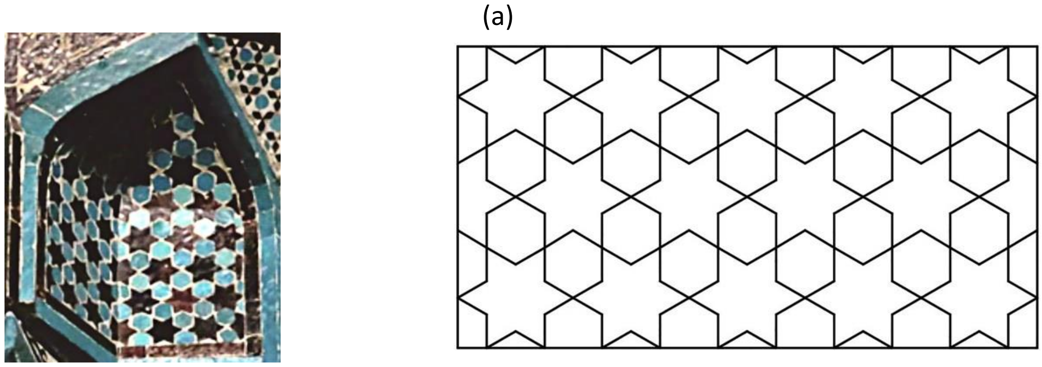
(a) (b)

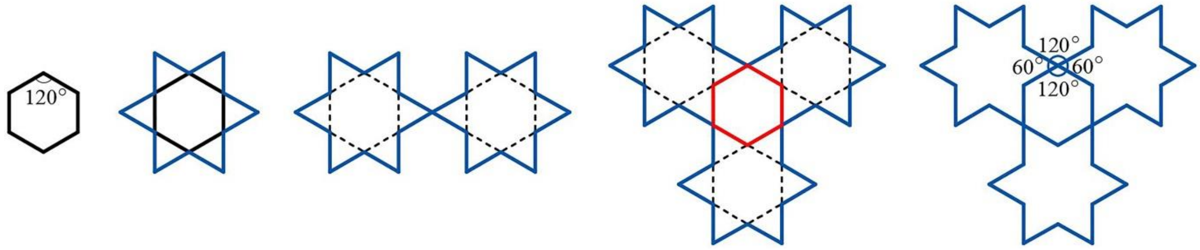
Fig. 6: (a) South facade of the mosque (Karakuş, 2021), (b) plan of the mosque (Erarslan, 2021)

The crown door with muqarnas continues the Seljuk crown door tradition with its side niches and decorations. The minaret, located to the right of the crown door, has a single balcony and is adjacent to the door. The hexagonal brick masonry body part tapers upwards and rises. The columns were enriched with muqarnas caps. The decorations on the consoles and beams show the originality of the building (Çilek, 2020). The walls of the mosque were covered with mosaic tiles. The building's prayer-hall (*harim*) is accessed through a pointed arched area covered with mosaic tiles. The prayer-hall is supported by 42 wooden-columns standing on 6 rows of stone pedestals and is divided into 7 sections (Erarslan, 2021). The mihrab and minbar, the most important part of the prayer-hall, are located at the south end of the middle aisle. The mihrab has mosaic tiles and is muqarnas. To the right of the mihrab is the wooden minbar, which is an example of Seljuk woodwork. Geometric shapes can be seen on a minbar made of walnut wood with the *kündekari* technique (Çilek, 2020). The mihrab and minbar sections where tessellation configurations are created are analyzed in Fig. 7.



(a)





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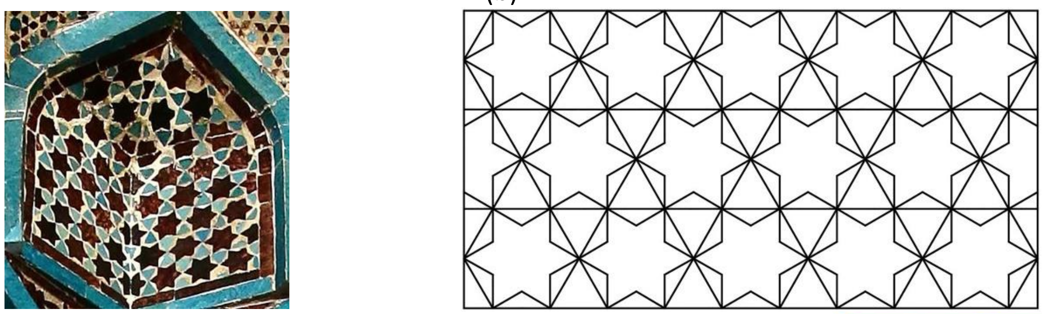
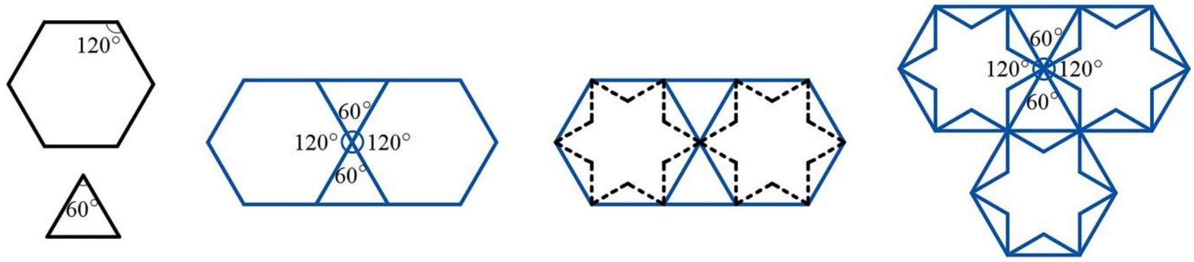
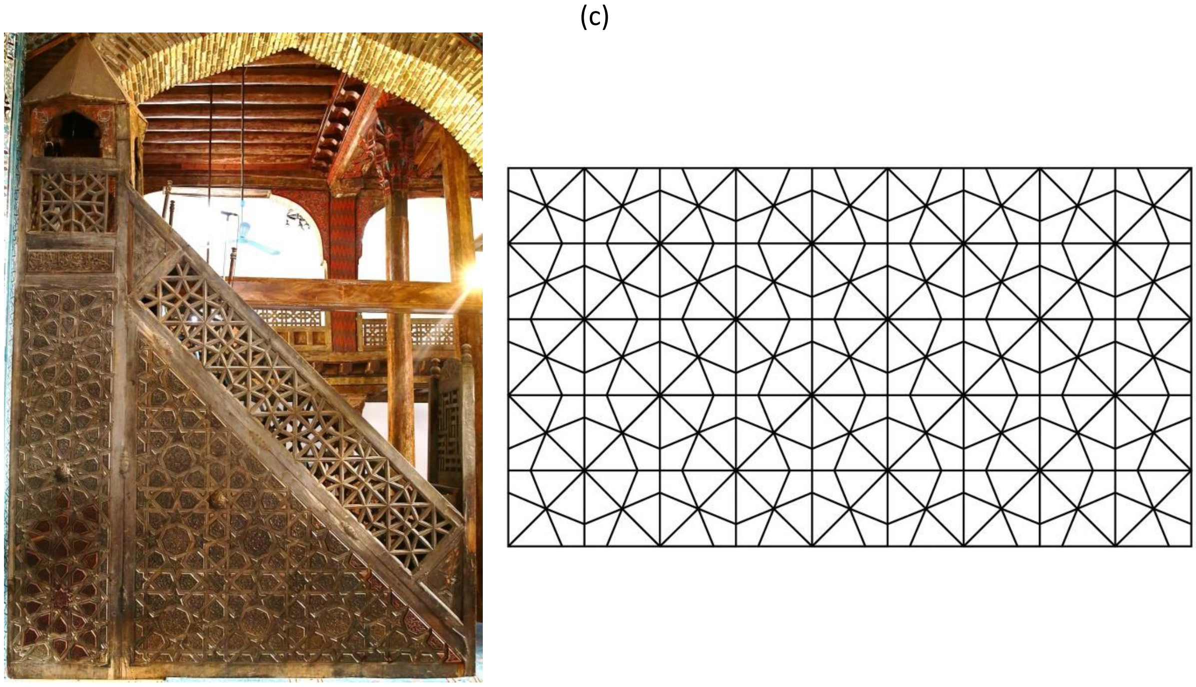
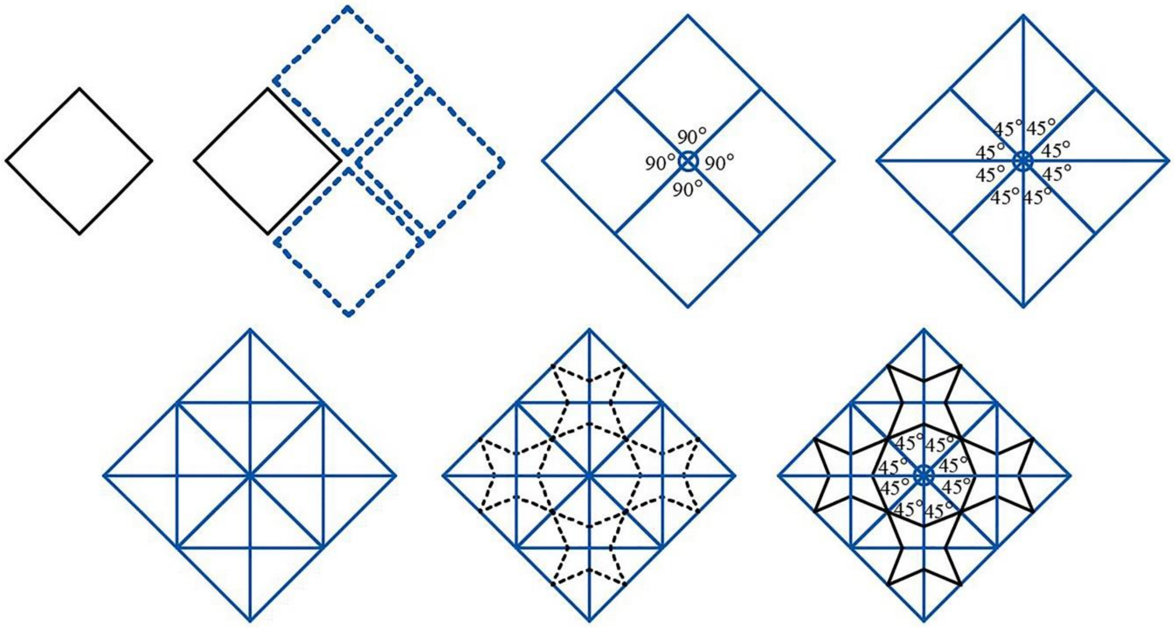




(c)



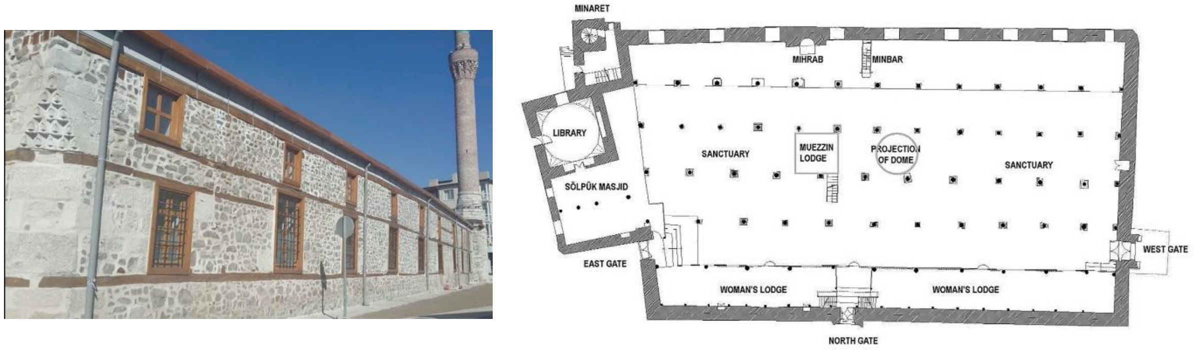


(d)

Fig. 7: (a), (b), and (c) Geometric analysis of the muqarnas details in the mihrab of the Beyşehir Eşrefoğlu Mosque, (d) geometric analysis of the railing detail on the minbar part of the mosque

Sivrihisar Ulu Mosque

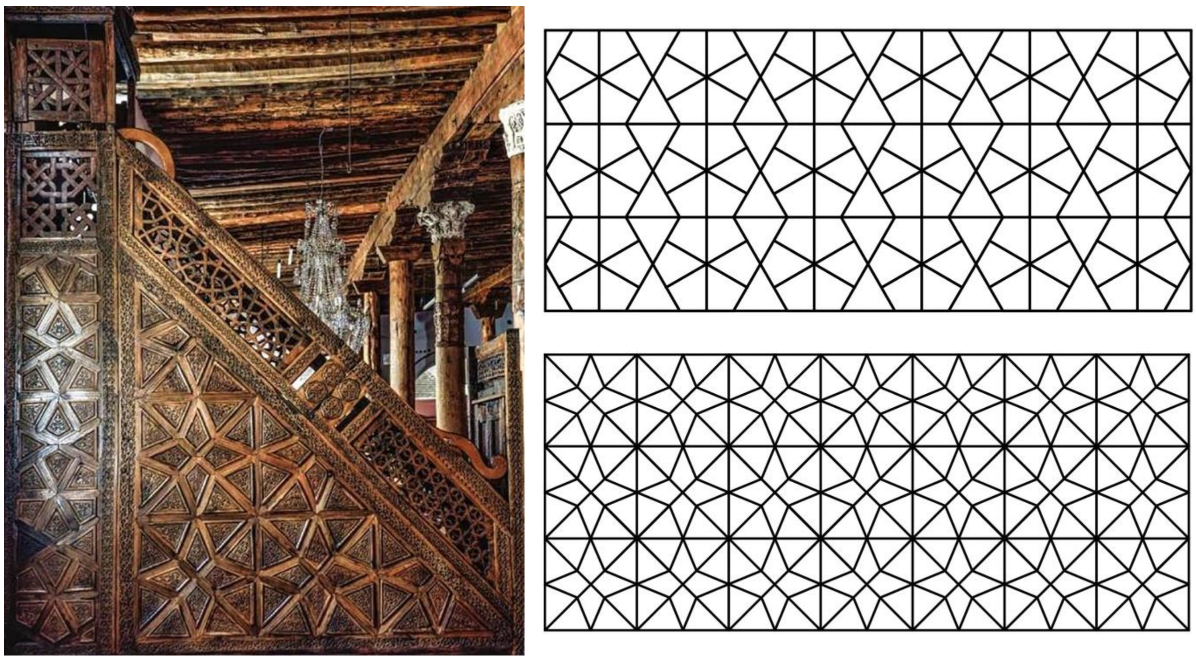
Located in Eskişehir, the mosque is one of the oldest surviving wooden-columned mosques in Anatolia. Although the exact date of construction is not known clearly, it is seen that it was built by Emir Cemaleddin Ali Bey in 1232 according to the inscriptions found in the building (Karakuş, 2021). The five inscriptions and documents on the mosque indicate that repairs were made in the building in 1232, 1274, 1409, 1440, and 1778. There are various documents from 1837 to 1925 in the Ottoman Archives and General Directorate of Foundations. Some of these documents are about the repair and demolition of the mosque. It is reflected in the documents that the most repaired part of the mosque is the roof (Koç, 2022). The mosque took its final shape in 1274 and 1275 with the interventions of Emînüddin Mikail Bey, son of Abdullah, who was a assistant of Gıyaseddin Keyhüsrev. A minaret was added to the building in 1409 and 1410. The base of the minaret was built of cut stone. The body of the minaret is octagonal and was built from stone and brick building materials. The lower part of the balcony in the minaret part is muqarnas. The mosque, which has a courtyard on the western façade, is applied on a slightly sloping land from north to south. The building, which was built using rubble stone with a masonry construction system, has a rectangular plan and measures 26.60*50.90 meters (Fig. 8). The structure was originally built with a flat earthen roof. In later periods, a sloping gable roof was built and covered with copper building material. There are entrance gates in the north, east, and west directions of the mosque (Karakuş, 2021).



(a) (b)

Fig. 8: (a) South facade of the mosque (Akyol, 2019), (b) plan of the mosque (Koç, 2022)

On the eastern facade of the mosque, there is a plain, marble jamb, rectangular, double-leafed wooden door. There are two repair inscriptions written on the marble above the door. Cut stone was used on the corners, facades, and door-window edges. At other points, there were rubble stones supported by wooden beams. There were two windows on the east and west facades and nine windows on the south facade. The ceiling section is supported by a total of 67 wooden-columns, 62 on the mosque and 5 on the masjid side. The decorations on the two columns in front of the minbar are different from the other columns. Other columns have similar ornaments and decorations (Koç, 2022). The prayer-hall is supported by 63 wooden columns parallel to the mihrab wall. Wooden columns were placed on reused marble plinths. There are decorated or undecorated wooden or reused spolia stones on the columns. The mihrab of the mosque is made of plaster and was built during the repair in 1339–1340. The minbar of the mosque was made of walnut wood using the *kündekari* technique. It is thought that this minbar, which was determined to belong to the years 1245–1246, was brought here from the Sivrihisar Kılıç Masjid, which was destroyed in 1924 (Karakuş, 2021). The minbar section where tessellation configurations were created is analyzed in Fig. 9.



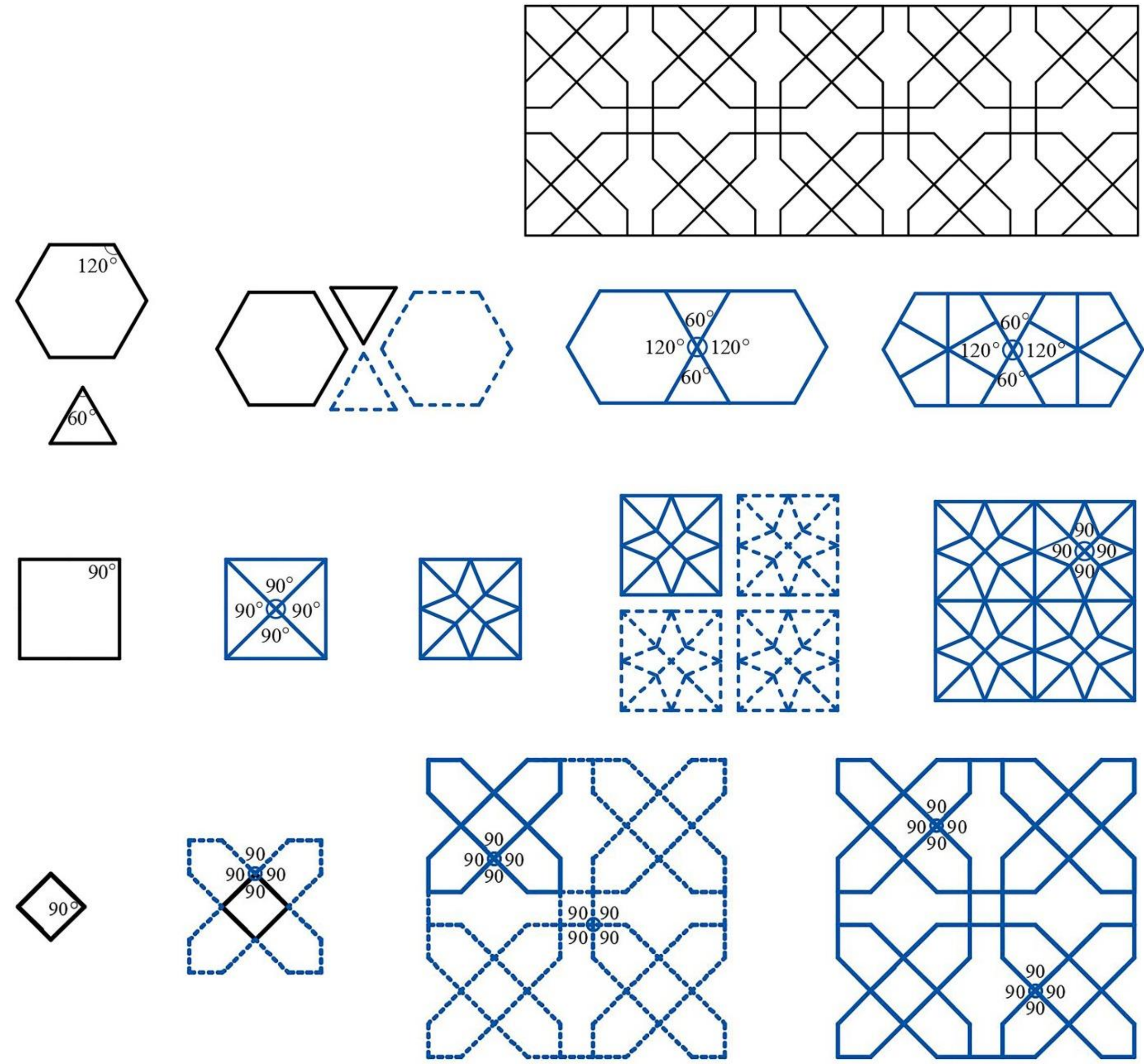
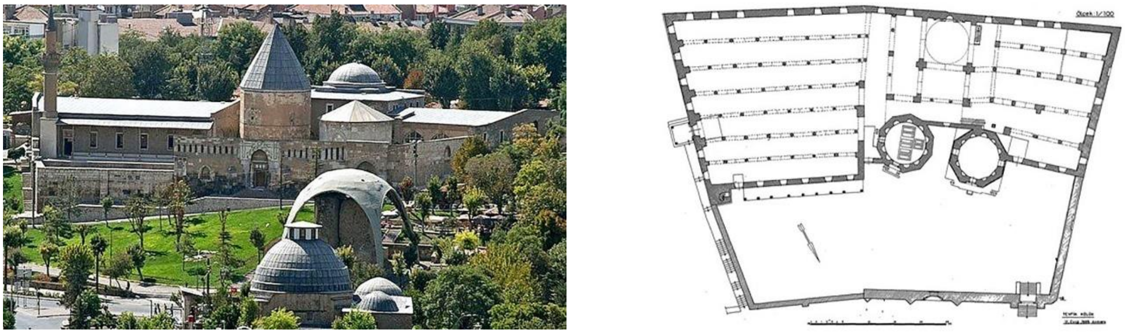


Fig. 9: Geometric analysis of tessellation configurations in the minbar section of the mosque

Konya Alâeddin Mosque

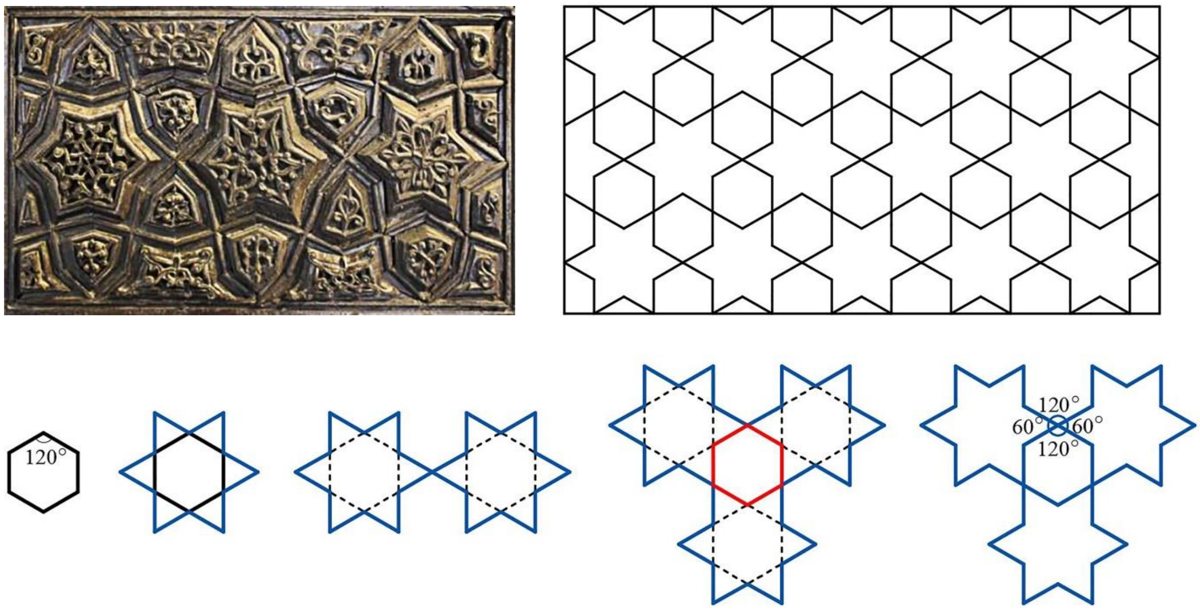
The Alâeddin Mosque, which belongs to the Anatolian Seljuk Period and is located in Konya, is one of the largest and oldest mosques. It is located on the upper side of Alâeddin Hill in Konya (Karabörk et al., 2015). The mosque, which was built in 1220, was completed in 1221 and was opened by Sultan Alaeddin Keykubat I. The building, which spreads over a wide area, consists of a courtyard and a prayer-hall (Detseli, 2018; Gökçe et al., 2016). There are two *kümbets* in the octagonal and decagonal geometric shapes side by side in the courtyard. Its construction took more than sixty-five years. There are traces of four Seljuk sultans and many craftspersons during the construction phase. The mosque has three gates, one in the east and two in the north, and there are two tombs in its courtyard. Seljuk period tiles were used on the dome and mihrab of the mosque. There are nearly twenty inscriptions written on different materials at different points of the building. The inscription on the minbar door, which mentions the oldest date, shows that the construction of the building was started during the govern of Sultan Mesud I (1116–1155) (Detseli, 2018; Gökçe et al., 2016). The mosque has undergone extensive repairs in different periods and has largely lost its original material. In the northern corner of the mosque, there is a minaret with 29 stone steps and made of bricks (Arıtan, 2020). The appearance and plan of the mosque are given in Fig. 10.



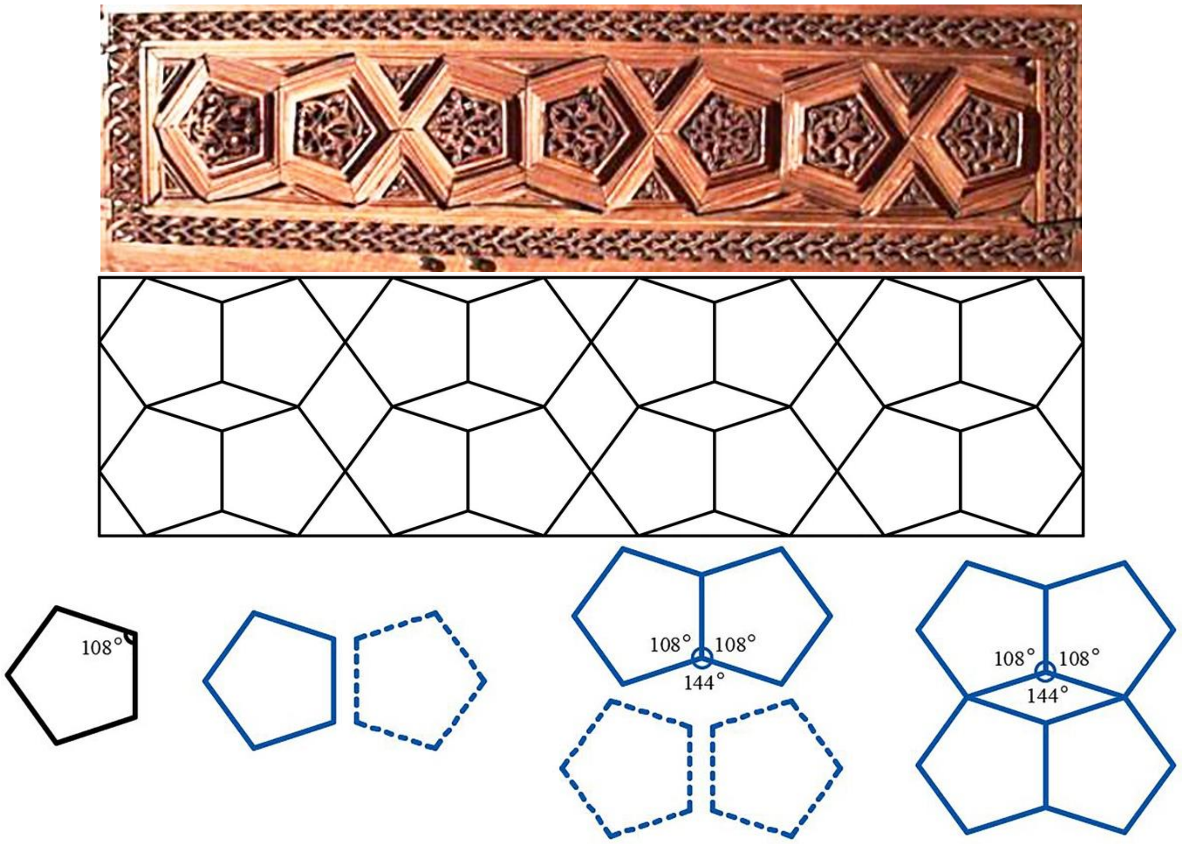
(a) (b)

Fig. 10: (a) Konya Alâeddin Mosque, (b) plan of the mosque (Detseli, 2018)

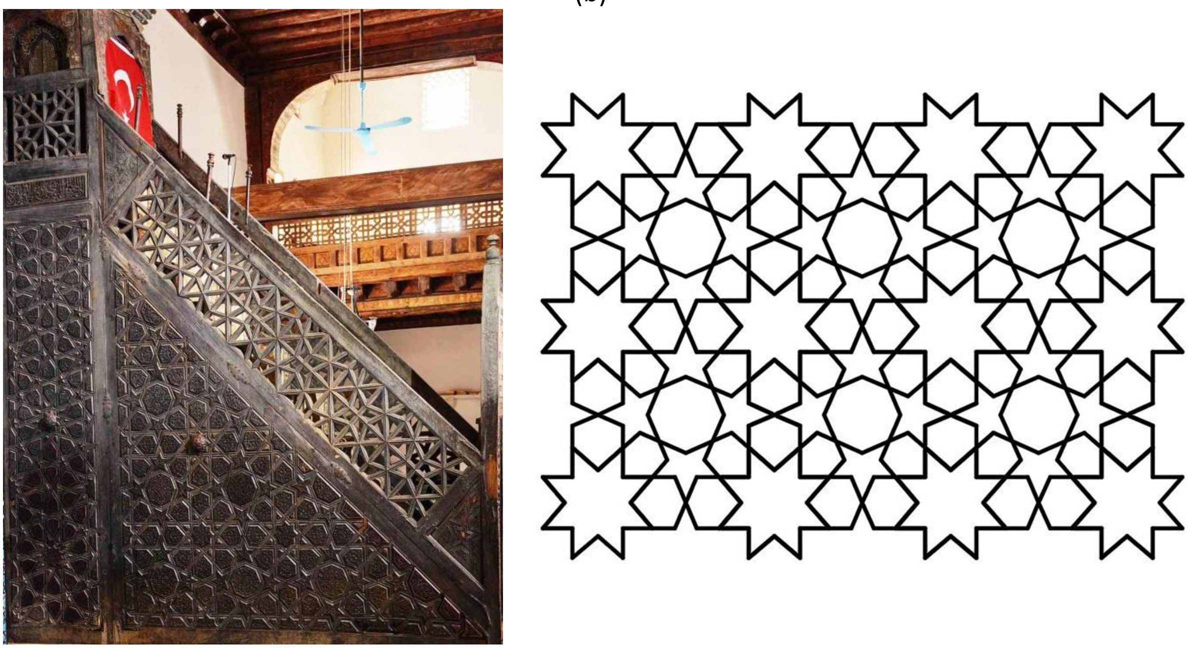
Walls made of cut stone surround the Alâeddin Mosque from north to east and west. 8 inscriptions are written in the Seljuk period on the marbles in the ornate niches on the walls. The length of the north wall is 59.7 meters and there are 21 windows, 6 inscriptions, and 3 doors, 2 of which were closed later, on this wall. There are 41 reused spolia stone and marble columns, mostly from the Byzantine period, in the irregular rectangular prayer-hall of the mosque, which is entered through the door on the east facade. The columns support the load-bearing system with the connection of the arches. The width of the mihrab, which has tile workmanship, is 6.05 meters. In the repair made in 1891, the lower part of the mihrab was completely replaced and a 4.40-meter-wide white marble mihrab was built. To the right of the mihrab, there is an interlocking minbar made of ebony wood, made with the *kündekari* technique (Arıtan, 2020). The minbar, which is one of the well-known wooden works of the Anatolian Seljuk period, was built in 1155 according to the inscription on it. The height of the minbar is 420 centimeters, its length is 360 centimeters, and the pointed arched opening (door) measures 71.5*125 centimeters (Detseli, 2019). In Fig. 11, the tessellation configurations in the minbar section of the mosque were analyzed.

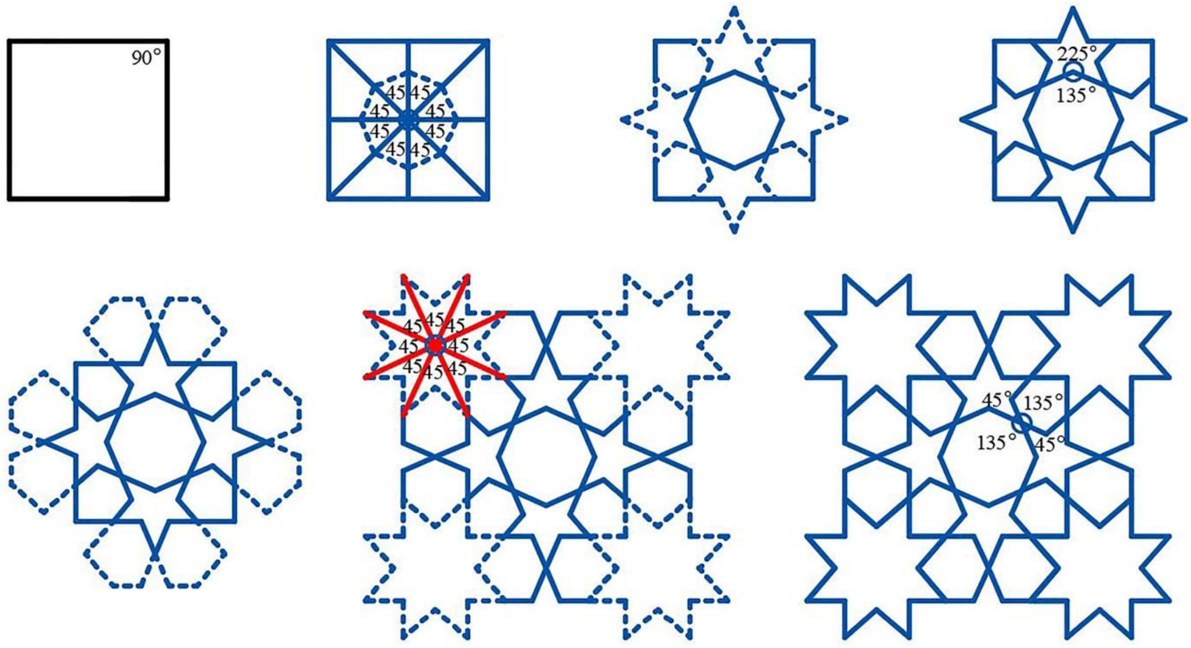


(a)



(b)





(c)

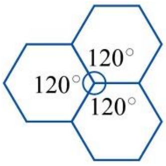
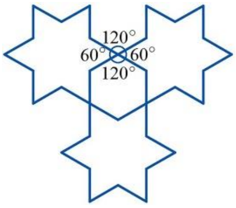
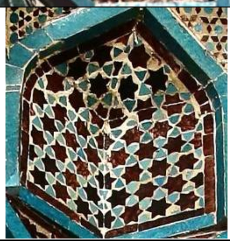
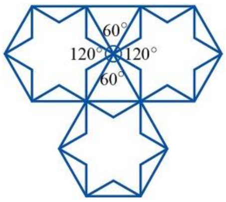
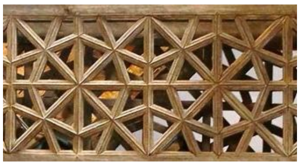
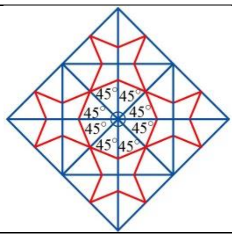
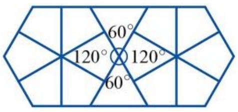
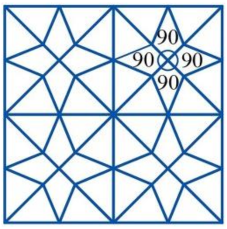
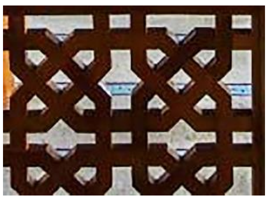
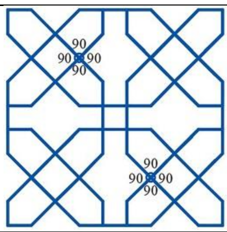
Fig. 11: (a) Analysis of the wooden geometric detail on the door of the minbar (b) analysis of the geometric detail on the door leaf of the minbar, (c) analysis of the geometric detail in the triangular part of the minbar

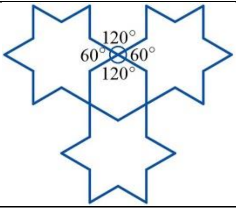
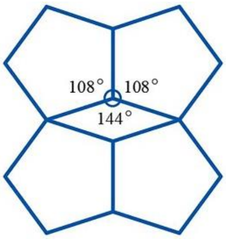
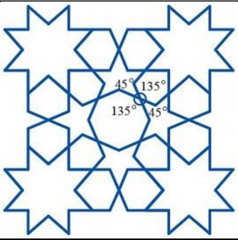
Findings and evaluations

Tessellation module, geometric shape, tessellation module formation and mathematical sequence parameters of Beyşehir Eşrefoğlu Mosque, Sivrihisar Ulu Mosque and Konya Alâeddin Mosque buildings, where tessellation configurations differ, were investigated. By looking at the parameters in Table 2, the relationships between the Islamic buildings were observed. It has been determined that there are different geometric variations in the mihrab and minbar sections of the marble and wooden-columned mosques. The formation mechanisms of tessellation configurations were examined according to different parameters and evaluations were made with the information obtained. The most used geometric shapes in terms of tessellation module are hexagon and star polygon. The plan of the Dome of the Rock (Kubbetü's Sahra), which has an important place in Islamic architecture, was also obtained from eight-pointed stars. Star polygon is frequently seen in ornaments and decorations. Considering that some tessellation patterns are derived from the hexagonal geometric shape in star polygons, it is seen that the hexagonal geometry is important in mosque buildings.

It is known that a regular hexagonal geometric shape consists of 6 equilateral triangles. For this reason, it can be proved that the triangular geometric shape is the basic geometric shape in tessellation configurations. The hexagonal geometric shape was found in the mihrab section of Beyşehir Eşrefoğlu Mosque, the minbar section of Sivrihisar Ulu Mosque, and the minbar section of Konya Alâeddin Mosque. Notably hexagon geometry is also used in all tessellation configurations where the star polygon is used. After triangular and hexagonal edge connections, quadrilateral and octagonal geometric shapes come to the fore. In Islamic building examples, it has been seen that quadrilateral geometry is obtained from geometric shapes such as square and deltoid. Quadrilateral geometry stands out on the minbar railings of Beyşehir Eşrefoğlu Mosque, on the minbar of Sivrihisar Ulu Mosque, and on the door leaf of the Konya Alaeddin Mosque. It has been determined that translation movement is used most in the creation of tessellation modules. In some of the tessellation configurations, it was determined that the reflection and rotation movements were used together with the translation movements. In addition to these, angle-edge connections completing 360 degrees are given in tessellation modules. Depending on the angles, mathematical sequences were created with the number of sides of the geometric shapes in the counterclockwise direction.

Table 2: Properties of mosques to which tessellation configurations are applied

	Geometric pattern	Tessellation module	Geometric shape	Tessellation module formation	Mathematical sequence
Beyşehir Eşrefoğlu Mosque			Hexagon	Translation	6.6.6 (6^3)
			Star polygon and hexagon	Translation	12.6.12.6 ($12^2.6^2$)
			Star polygon, hexagon and triangle	Translation and reflection	6.3.6.3 ($6^2.3^2$)
			Square and octagon	Translation and reflection	3.3.3.3.3.3.3.3 (3^8)
Sivrihisar Ulu Mosque			Hexagon, triangle and quadrilateral	Translation and reflection	4.3.4.3 ($4^2.3^2$)
			Square and octagon	Translation	4.4.4.4 (4^4)
			Square, pentagon and dodecagon	Translation, reflection and rotation	4.5.12.5

Konya Alâeddin Mosque			Star polygon and hexagon	Translation	12.6.12.6 ($12^2.6^2$)
			Pentagon and quadrilateral	Translation and reflection	5.5.4
			Hexagon, octagon and star polygon	Translation and reflection	6.10.8.10

Conclusion

The concept of tessellation in the architectural design process is an application in which the science of geometry is included in the field of architecture. Versatile planning can be made with the geometric patterns transferred to architectural designs. With this method, which also helps provide building diversity, it is possible to switch from small-scale production to large-scale construction from the building element to the formation of the building itself. Developments in building technology and methodology have also opened the door to geometric freedom. The ability to shape different structural formations with building materials brings the integration of geometric freedom. There are also tessellation patterns in complex building systems. It can be said that different triangle shapes also create different tessellation configurations. Obtaining different tessellation patterns even in a single geometric figure indicates the depth and expansionist potential of the tessellation method. In tessellation configurations, in addition to triangular, quadrilateral and hexagonal geometries, combinations of these basic geometries and different formal organizations derived from these geometries can be used. The diversity of regular and semi-regular tessellation patterns is seen in the Islamic architectural examples. The examined mosques have also emerged with the integration of geometry's understanding of order. This understanding of order is also seen in Beyşehir Eşrefoğlu Mosque, Sivrihisar Ulu Mosque, and Konya Alaeddin Mosque. At this point, it is foreseen that the architectural production process on a wide scale can be considered multidimensional according to the infinite order of tessellation and geometry. In future studies, it is thought that with the increase in the diversity of mathematical calculations and digital designs, the tessellation method will develop, and geometric studies and applications will reach a different dimension.

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