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# Crystallographic Patterns in Turkish-Islamic Architecture with The Perspective of The History of Mathematics and Crystallography

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**Abstract**

Our study is an introduction to the underlying reasons why the concept of crystal (crystallographic) pattern, which emerged from the union of mathematics and crystallography in the 20th century, is a frequently used ornamental element in Turkish-Islamic architectural works dating back about 9 centuries. Our aim is the geometric ornaments, which are generally handled by art historians; the aim is to underline that it is an area that needs attention for historians of mathematics, crystallography and architecture. We will try to reveal the emergence of crystal patterns, one of the patterns used by Turkish-Islamic architecture for centuries to cover many surfaces, in the fields of mathematics and crystallography, with the perspective of the history of science, in the context of two basic theories. The first of these theories is symmetry, and the second is group theory, which provides the basis for the mathematical explanation of symmetry. The examples of crystal structures, which are the subject of the science of crystallography, which developed with the help of new technologies discovered after the mathematical possibilities provided by symmetry and group theory, were applied centuries ago in Turkish-Islamic architecture will be emphasized.

In this way, it will be noticed that the geometrical ornaments used in the artifacts from the Turks and the formation principles of the crystals are the same. These patterns, which were produced in a period when symmetry groups or crystallography were not known at all, were used as a kind of cultural element, especially by the Karakhanids, Seljuks, Anatolian Seljuks…etc. It is surprising that it is seen in all geographies that Muslim Turks have reached. Considering the history of mathematics and crystallography, it is understood that the crystal patterns used in architecture are a manifestation of the knowledge of Turkish-Islamic civilization in many sciences, especially geometry.

**Keywords:** Geometric Ornaments, Crystallographic Patterns, Crystallography, Symmetry Groups, The History of Mathematics, The History of Turkish-Islamic Architecture.

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## I. Introduction

The construction is the way the entity creates variety. Everything that constitutes nature has gradual construction, and each stage has core atoms. The construction of geometric ornaments of Turkish-Islamic works is based on the periodic repetition of a basic motif for the same purpose as the crystal formations in nature. It is very interesting that ancient masters used mathematical principles centuries ago that coincide with crystal formations in nature.

We encounter three basic elements in the decoration of Islamic architectural works: calligraphy, plants and geometric patterns. The predominant use of geometric ornaments on the walls, ceiling and other parts of the building can be basically based on two reasons. The first is that Muslims do not welcome the drawings of animal and human images in a place of worship. The second is the aesthetic expression of the relationship between the unity of God and the multiplicity of the creature through the geometric ornaments in the works of architects and artists. Especially self-repeating periodic patterns are the expression of eternity, the most important element of Islamic metaphysics. This idea paved the way for mathematics to be a language of art as well as being the language of science. It can be said that the publications started with the publications of E. Prissed 'Avennes after 1850, who was influenced by the delicate geometric ornaments of Islamic works he encountered in Cairo in 1827. Later, in 1879, Bourgoin wrote a work titled *Les Elements de L'Art Arabe*, which contained many sketches of geometric ornaments (Bourgoin 1879, 49-190). With his article titled "The Drawing of Geometric Patterns in Saracenic Art" in 1925, E. H. Hankin was the first person to draw attention to the polygonal technique in the creation of patterns by emphasizing the symmetries found in geometric patterns (Hankin 1925, 4-9). In Turkey in 1979 Yıldız Demiriz's book of *Decorations Early in the Ottoman Architecture (Erken Dönem Osmanlı Mimârisinde Süsleme),* in 1982 Selçuk Mülayim's work titled *Geometric Ornaments in Anatolian Turkish Architecture* (*Anadolu Türk Mimârisinde Geometrik Süslemeler)*, 2013 Hüseyin Şen's article titled "Geometric Patterns in Islamic Art" (İslâm Sanatı'nda Geometrik Desenler) are the main studies on geometric patterns. When it comes to the recent period, in 2017 Jay Bonner represents an important advance in the analysis and classification of Islamic geometric patterns. In his work titled *Islamic Geometric Patterns*, Bonner concentrated especially on the structures built during the Seljuk period and focused on the unsystematic design methods used in the creation of geometric models.

Geometric ornaments have become the subject of study in different disciplines such as chemistry, crystallography, biology, physics and mathematics, as well as art historians for the last forty years. In 1986, Azeri crystallographer S. Khudu Mamedov brought the concept of crystallographic patterns to the scientific literature by embodying the similarities between the crystal structures in crystallography and the geometric ornaments in Turkish-Islamic works. Chemist Hacali Necefoğlu, a student of Memmedov, also has studies on crystallographic patterns.

The claim that the formations (construction) of geometric patterns and crystal structures have common mathematical principles revealed the fact that the processes of creating these patterns by ancient masters should be examined. There are two possibilities for the masters who draw geometric patterns with the help of ruler and compass. First, they have a good knowledge of geometry and created these patterns themselves. Second, they got help from brilliant mathematicians who lived in the same period. In both cases, they successfully applied the rules of symmetry, which were expressed mathematically in the late 19th century, in their works hundreds of years ago. In order to clarify how this is possible, we will try to explain the adventure of crystallographic patterns that have been used by Turkish-Islamic architects to tile for centuries, with the perspective of the history of science, in the fields of mathematics and crystallography, in the context of two basic theories. The first of these theories is symmetry and the second is the group theory, which provides a basis for mathematical explanation of symmetry. After the mathematical advantages provided by symmetry and group theory, the science of crystallography, which developed with the help of newly-discovered technologies, revealed the formation of crystal structures more clearly. In this study, Turkish-Islamic buildings of art in which crystal structures are used as decoration elements will be emphasized.

## II. Background

### *A. The Emergence of Symmetry and Group Theories*

The human mind likes symmetry. Symmetry is a concept that emerged with the development of the idea of being (staying) the same. In other words, it is the preservation of the properties of a given object such as shape and position without deterioration. Mathematicians came to symmetry groups, and hence group theory, as a result of debates about whether $5^{th}$-degree equations have solutions. Before talking about symmetry groups, it would be appropriate to briefly mention the group concept that emerged in the search for the solution (or insolubility) of $5^{th}$ degree equations. Although the group theory is an indispensable element of contemporary mathematics, it is also important due to its relationship with symmetry.

The first effective solution to the $5^{th}$-degree equation came from the famous mathematician J. L. Lagrange (1736-1813). It was worth studying how algebraic expressions behave structurally when the order of the solutions of a $3^{rd}$-degree equation was changed. According to Lagrange, the order of the solutions in question did not matter. Any symmetric expression could be written in terms of the coefficients of the equation. For the solution, it was possible to reduce a tertiary equation to quadratic by using partially symmetric functions (Stewart 2017, 205). Lagrange's method was applied to the solution of $4^{th}$-degree equations, but it turned out to not work for $5^{th}$-degree equations. Afterwards, even though C. F. Gauss (1777-1855) tried to solve this issue, he did not succeed. It is Lagrange's progress in group theory that permutations of roots should be taken into account in the study of algebraic equations. In this way, the concept of abstract group is one step closer. It was proved by J. R. Argand (1768-1822), Gauss and Lagrange that every equation has a real or a complex solution, and thus the issue of solving $5^{th}$-degree equations is guaranteed. Interestingly, P. Ruffini (1765-1822), who opposed this view, made the claim in his *General Theory of Equations* in 1799 that it is not possible to solve high equations of the $4^{th}$ degree algebraically (Cajori 2014, 395). This counter-approach spread the idea that radical expressions are not useful in solving $5^{th}$-degree equations. Ruffini also claimed that permutations can be combined between each other, leading to a second advance in the group concept after Lagrange (Kökcü 2019, 115).

The next step came from Niels Henrik Abel (1802-1829). Abel published an article in 1824, confirming Ruffini's thesis, about the absence of algebraic solutions for equations higher than $4^{th}$ degree (Boyer 2015, 563). Abel suggested that we can obtain new ones in two ways, using existing quantities. The first way was to combine a quantity such as x using multiplication, division, addition and subtraction operations. For example, expressions such as $3x$, $2x + 5$, $x^2$ could be obtained by four algebraic operations applied to x. The second way was to make use of radical expressions. What Abel did in his article was simply to complete the missing step Ruffini had left. Abel argued that any added function other than the coefficients of the equation would not lead to a solution. Abel proved the insolubility of $5^{th}$-degree equations by applying his arguments to the symmetric function theorem (Kline 1982, 294).

After Abel, it was necessary to clarify which equations could or could not be solved with radical expressions. It was the mathematician Evariste Galois (1811-1832) who authored the work that most influenced those after him on this subject. Galois did not confine himself to solving only $5^{th}$-degree equations and began to think about the solutions of all equations. Galois embarked on a search to determine which of the equations could be solved with the help of radical expressions. Like his predecessors, he paid attention to the symmetry, the behavior when the solutions are permuted, based on the algebraic solution. According to Ruffini and Abel, an expression in solutions should not always be symmetrical but could be semi-symmetrical. According to Galois, if any two of the permutations that organize some expressions in the solution set are multiplied with each other, the resulting expression also regulates the permutation (Stewart 2017, 213). Galois called this permutation system "group". According to Galois's conclusion, $5^{th}$ equations were unsolvable because they had an incorrect symmetry.

We can use a classical example to explain the relationship between symmetry and group theory more clearly. Suppose that you have a square and a circle. First, let us state that these are ideal squares and circles, as Plato meant. Let's ask if the square or the circle is more symmetrical. Before considering the answer to the question mathematically, it will be suggested intuitively that the circle is more symmetrical. But when proof of this is requested, we must enter into other inquiries. First, let's start with the square and list the properties of the square. A square has four equal sides and four equal angles. If the intersection point of the diagonals of the square is accepted as the center, we see that the square takes the same shape without distortion when we rotate it $90^0$ around this center. So, the square comes on itself every $90^0$ around its center and remains intact, preserving its initial length and angle values. We have seen that the two most important elements of Euclid's geometry, angle and length, do not undergo any change for the square on which we apply rotational symmetry. So, as a result of rotating a square in ($90^0$, $180^0$, $270^0$, $0^0$) degrees, its shape remains intact, and these rotation angles of the square give their symmetries (Kökcü 2019, 118). If we denote the set of symmetries that the square has with S, then S = {$90^0$, $180^0$, $270^0$, $0^0$}. We can list the relationships between the elements of the set S as follows.

**1.** The composition of the elements belonging to the set S is the element of the set S, so the set S provides the closed property. The answer to the question of what happens if a square is rotated $270^0$ and then $180^0$ can be answered by taking the composition of these two movements. The sum of the angles is $270^0 + 180^0 = 450^0$ and since the full angle of a circle is $360^0$, $90^0$ is obtained by taking the mode of 450 relative to 360 which is called addition module 360. This means that there is no difference in consequence between rotating the square $90^0$ and rotating $450^0$, and $90^0$ is an element of the set S.

**2.** Set of S has the associative property. This means that the two symmetries can be combined into (90 + 270) + 180 = 90 + (270 + 180).

**3.** S set has the identity element property, and the identity element is 0 degree.

**4.** Set of S has an inverse element property. To illustrate this, if we rotate a square $180^0$ clockwise first and then $180^0$ counterclockwise, we see that the square returns to its original position (Frenkel 2015, 36-40).

The (S, +) group has been obtained thanks to the 4 properties mentioned above that the set of S has by rotation. Group theory deals with the common behavior of these structures under the operations applied to algebraic structures. If the members of group S are interchangeable, then the S group is called an Abelian group. Now let's see if the same properties apply to the square. If the circle is rotated around its center, it is not possible to understand intuitively if it has changed. Since the circle doesn't have any corner, it actually has infinite symmetry. Hence it forms an infinite but closed group.

### *B. The Birth of Crystallography*

The symmetry and group concepts that emerged with the discussion on the solution of higher degree equations in the field of mathematics were also closely related to mineralogy, which is one of the applied sciences. Rene Just Haüy (1743-1822), whose field was mineralogy, was the person who determined the geometry in the shapes of crystals. Actually, it was completely the result of an accident. While examining the pieces of calcite, Haüy noticed that the pieces that had accidentally fallen to the ground were splitting straight at fixed angles. As a result of this, Haüy broke more pieces and saw that whatever the original shape was, the broken pieces always had a rhombus shape. He revealed the geometric structure theory of crystals as a result of detailed experiments he had done. His theory was mainly concerned with two things: first, the laws of decay and stability of the angles, second, their relationship to the primary forms or nuclei of crystals (https://www.britannica.com/biography/Rene-Just-Hauy, Accessed February 10, 2021). This meant that crystal surfaces always had properly selected axes (Burckhardt 1988, 19). After expressing his theory in this way, Haüy classified the structures of minerals. Thus, he became one of the pioneers of crystallography.

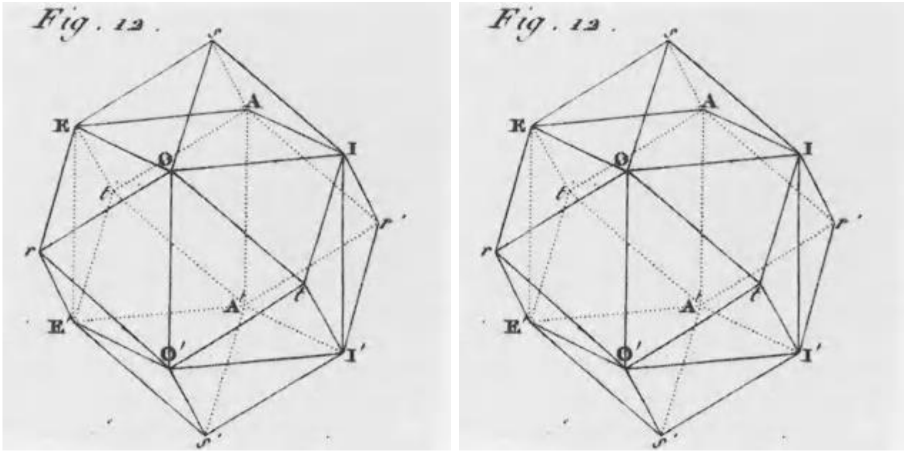


Fig. 1: Haüy's drawings (Burckhardt 1988, 19)

Another pioneer in the field of crystallography is the French physicist Auguste Bravais (1811-1863). Dealing with the forms of the outer and inner structures of crystals, Bravais published two articles in 1848 and 1849. The subject of his articles was on the geometric structures of crystalline formations such as quartz and diamonds. The atomic patterns of minerals, especially salt, have very special geometric forms. These substances are compositions formed by different atoms coming together. That is, they are made up of patterns that repeat throughout a crystal. The atoms that make up a crystal form a uniform three-dimensional lattice. Bravais comprehensively analyzed the geometry of the molecular polyhedral in his work titled *Études Cristallographiques*, published in 1866 (Kline 1982, 294). According to Bravais, there are 7 different symmetries in the case of a 3-dimensional lattice for a crystal. 7 different types of symmetry are as follows: cubic, hexagonal, rhombohedral, tetragonal, orthorhombic, monoclinic and triclinic (Pitteri & Zanzotto 1996, 835).

The first scientific application of group theory that emerged in the field of mathematics was used in the classification of all possible crystal structures. Using group theory, Camille Jordan (1838-1922) classified the basic types of motion of Euclid's solids. Jordan was struck by the crystal symmetry inherent in crystals, demonstrated by Bravais. This effect was so great that Jordan generalized Bravais's work with his work published in 1868-1869 (Stewart 2017, 214). Jordan published a short note in *The Comptes Rendus* in Paris in 1867. The note was titled "Sur les groups de mouvements", which means on motion groups. That is, it was about the exact determination of all possible motion groups of solid bodies in three-dimensional Euclid space (van der Waerden 1985, 119). There were two theories on the basis of Jordan and Bravais's work: group and symmetry theories.

Jordan was aware that there is a geometric relationship between the basic types of motion of a solid body and group theory. Jordan only dealt with closed groups and first identified all the translation and rotation groups. According to Jordan, basic solid movements on the plane (in two dimensions) are possible in four ways; rotation, translation, reflection (mirroring) and translational reflection. In addition, in three-dimensional space, apart from these four movements, there is another movement known as the screwing movement, the translation of the object both around its own axis and along a fixed axis (Stewart 2017, 214).

Jordan lists ten types of motion in the translation groups, which are related to continuous translation in any direction and discontinuous translation in exact multiples of a fixed distance. When it comes to closed rotation groups, finding them does not seem as easy as translation. Discontinuous closed rotation groups are divided into five as cyclic, dihedral, tetrahedral, octahedral and icosahedral. Continuous rotation groups are three types: the group of all rotations with a center point such as 0, the rotation group about a fixed axis, and the rotation group about the same axis that is shifted around a fixed axis (van der Waerden 1985, 119-120). Jordan's list was incomplete, but he still managed to take his place among the pioneers in this field.

After Bravais, Wilhelm Conrad Röntgen (1845-1923) discovered X-rays in 1985. X-rays falling on the surface of a crystal structure with small incidence angles are fully reflected. X-rays are scattered by the parallel planes of the atoms in the crystalline structure. A large amount of scattering of the crystal reflected from the crystal is called refraction. The refraction of x-rays on crystals was first discovered in Munich in 1912 by Max von Laue (1879-1959) (Schwarzenbach 2012, 57). For this work, Laule was awarded the Nobel Prize in Physics in 1914. Thanks to the use of x-ray diffraction technology, two- and three-dimensional space lattice models were determined by crystallography. After 1912, the development of crystallography accelerated.

### *III. Use of Crystallographic Patterns in Turkish-Islamic Architecture*

Crystal structures are regular shapes that grow by repeating molecules or atoms without leaving any spaces between them. We mentioned earlier in Bravais's work that crystals have translational and rotational symmetries. Much more of what Bravais and Jordan later expressed with symmetry groups emerged in the field of crystallography with the use of x-rays after 1912. When we come to the 20th century, we have two basic forms about the construction of matter, which we have obtained with the opportunities provided by mathematics and crystallography. The atoms (molecules) that make up the matter are either arbitrarily formed in an amorphous order or in a crystalline structure.

Let us give the most given example in order to explain what their different construction forms mean for matters. Diamond and graphite are both composed of the element carbon. Although the substance and amount they are composed of are the same, the properties of the two substances are quite different from each other. Graphite can be crumbled while diamond is very hard and shiny. The difference between the two substances stems from the organization of their molecules (Mamedov 2002, 232). The most basic rule of creating diversity with the limited number of elements that exist in nature is based on the differences in the constitution of substances. Just as all sounds are obtained from seven notes in music, in nature, all substances can be formed with about a hundred elements. Design customizes the element and in the same way the element customizes the design. Even if the integrity cannot be given according to the amount of the things in the design, if the rules of the pattern it has are determined, it can be predicted how it will continue. That is, the structure does not consist of a random combination of parts. The central effect in determining the pattern is due to symmetry. Symmetry is not the reason for the arrangement of the design, but it is one of its most necessary measures. As in the examples of squares and circles above, there is less and more of symmetry. Since our subject is geometric ornaments, 2-dimensional symmetries on the surface are mentioned here.

Since ancient times, geometric patterns have been used as a decorative tool. However, the examples of geometric patterns that repeat each other in crystal structure, that is, without background and in certain periodic proportions, are most frequently encountered after the 11th century (Ekizler Sönmez 2016, 39). Crystal structures have both translational and rotational symmetry. The same pattern is obtained when a pattern with translational symmetry is rotated 360/2 = 180 for 2-fold, 360/3 = 120 for 3-fold, 360/4 = 90 for 4-fold and 360/6 = 60 for 6-fold. It is observed that patterns with 5-fold, 7-fold, 9-fold and 11-fold symmetries do not have translational symmetry (Ekizler Sönmez 2016, 40). If both translation and rotational symmetry are valid for a pattern, it means that there is a unit cell in which we can produce the pattern in question. If the unit cell remains intact when rotated around an axis by a certain angle, it has point group symmetry. If we express the angle of rotation as n 2Л, only when we use 2, 3, 4 and 6 for n, point group symmetry and translational symmetry work in harmony. When there are 5, 7, 9 and 11, the harmony is broken, and these symmetries are expressed as forbidden (Lu &Steinhardt 2007, 1106).

In the 1980s, patterns used in geometric ornaments began to attract the attention of chemists and crystallographers. The term crystallographic pattern was first introduced by Azeri chemist and crystallographer S. Khudu Mamedov in his article *Crystallographic Patterns* in 1986. Mamedov gathered the features of crystal patterns found in Turkish-Islamic architects in 5 categories.

1. The boundary of a crystallographic pattern ends with the boundary of the shape elements.

2. Pattern elements are placed on the surface as densely as possible.

3. The decorated surface has no background or a background is converted into the pattern element.

4. The variety of geometric shapes used in the pattern and the number of complex pattern principles are kept as low as possible.

5. In crystal patterns, symmetry is not used to create shapes, but symmetry occurs as a result of this situation when the pattern is formed (Mamedov 2002, 249).

The symmetries of these patterns with five properties are similar to the symmetry they have in the structures of natural crystalline materials.The structure of a quartz crystal can be a good example to demonstrate the similarity of the crystal pattern that a crystal has and the crystallographic pattern used in architecture. The stable molecule sequence of $SiO_2$ is as follows.

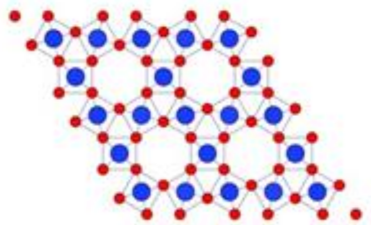

Fig. 2: Molecular organization of $SiO_2$

The vertical projection of $SiO_2$ to the surface will be the combination of the shapes formed by the connecting lines between them. If the figure is examined closely, it will be noticed that there is a hexagon in the middle and squares with a common edge around the hexagons and equilateral triangles that connect with other motifs. This symmetrical shape that repeats itself is among the most preferred hexagonal patterns in Turkish-Islamic art. When a plane or a curved surface is tiled with a geometric pattern without the need for another background, the concept of infinity, which is the most important element of Islamic art, is emphasized. An example of decoration in a similar style is located in the tomb of Daye Hatun (Green Tomb), which is located in the tomb of Mehmed I (d. 1421), which was built in the first quarter of the 15th century in Bursa.

**Photo 1:** Daye Hatun's Tomb, Green Tomb

(Photo http://www.semerkanddanbosnaya.com/portfolio/yesil-turbe/ Accessed on February 15, 2021)

The wall decorations of the Green Mosque, which is just opposite the Green Tomb, are in the form of a crystallographic pattern formed by the base of free radicals of $SiO_2$. It may not be immediately noticed that some of the patterns used in architectural works are associated with geometric motifs. The elements and internal organization of such models are reminiscent of two-dimensional crystals and their projections. Therefore, this type of decoration is called crystallographic pattern (Mamedov 1986: 511).

Mamedov divides crystallographic patterns into two as geometric motifs such as pattern elements and patterns made of kufic scripts (Necefoğlu 2017, 558). One of the oldest examples that can be given to kufic scripts is the Berde Tomb located in the Berde region of Azerbaijan. The history of the city of Berde goes back to the Scythian Turks. The tomb, located in the city, which was under the rule of the Seljuks and then another Turkish state, the Atabegs, was built by Ahmed B. Eyyub Nakhcivani in 1322 during the Ilhanid period (Çağlıtütüncügil 2008, 35). The number of Islamic art works, the masters of which are known, is very few. That's why, it is a great chance that the master of Berde Tomb is known.

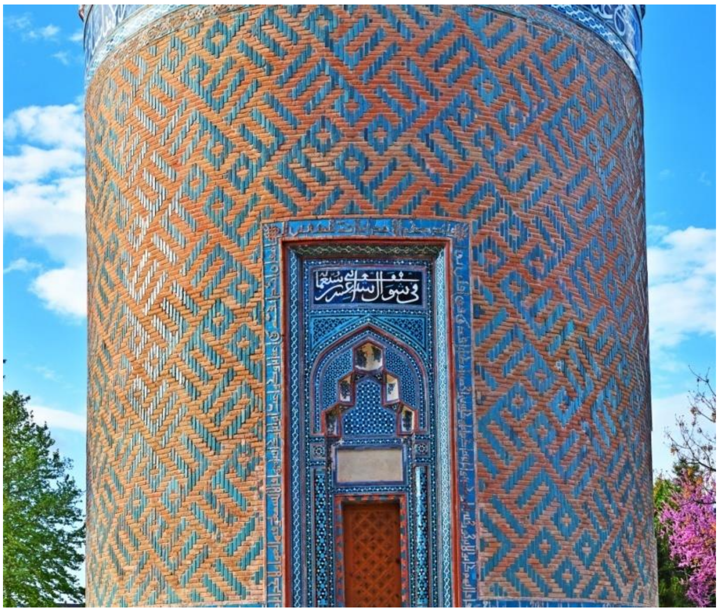

**Photo 2:** Berde Tomb (Photo https://az.wikipedia.org/wiki/B%C9%99rd%C9%99_t%C3%BCrb%C9%99si Accessed on March 16, 2021)

The outer surface of the tomb, which has a cylindrical shape, is tiled with a pattern obtained by making use of the Arabic word "Allah". There is a 45-degree angle between the unit cell covering the tomb and the cylindrical line. In this way, dynamism is given to the pattern. The master who made the flooring did not overlap the crystallographic writing pattern he placed on the cylinder surface and did not leave any space between them. One of the regions where crystallographic writing patterns are most popular outside of Central Asia is Anatolia. The words "Allah", "Muhammed" and "Ali" are the most preferred ornamental phrases.

According to Mamedov, the principles of formation of crystals and geometric ornaments used in Turkish artworks are the same. The earliest examples of these patterns are found in the works of the Qarakhanids, the first Muslim Turkish state. These patterns, produced in a period when symmetry groups and crystallography were not known at all, are thought to be the pattern language of the Turks as a kind of cultural element (Necefoğlu 2017, 558). Crystallographic patterns turn into a cultural identity code that can be traced as a cultural element that Turks have carried to different geographies. The first use of geometric patterns in Islamic art is mostly attributed to the Abbasids by art historians. Although complex geometric compositions are encountered in the Abbasids, especially between the years 750-960, it can be said that these decorations are still in development when compared to the

Qarakhanids and Great Seljuk Architecture. On the other hand, examples of geometric ornaments seen in the architectural structures of the Abbasids can be attributed to the Turkish population in them (Ekizler Sönmez 2014, 129).

The structures built by the Karakhanids with brick materials are the first examples of the Turkish-Islamic Architecture. Built in the 11th and 12th centuries, Talhatan Baba Mosque, Degaron Mosque, Arap Ata Tomb, Ayşe Bibi Tomb are among these structures. After the Karakhanids, especially during the Great Seljuks and Anatolian Seljuks, geometric patterns evolved into ornaments with extremely complex compositions.

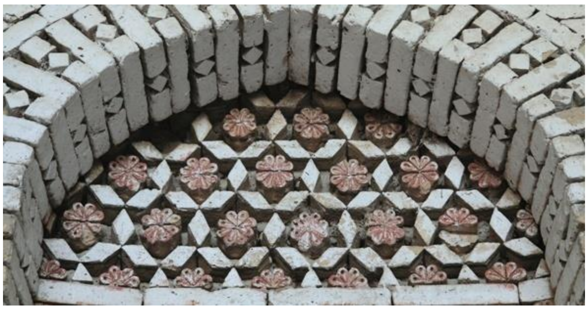

**Photo 3:** Crystallographic pattern decorations on the mihrab of Talhatan Baba Mosque, built in the late 11th century (Photo http://www.selcuklumirasi.com/architecture-detail/talhatan-baba-camii Accessed February 12, 2021)

Between the 11th and 15th centuries, states such as the Karakhanids, Ghaznavids, Seljuks, Atabegs, Ilkhanians and Timurids in which the majority of their rulers and people were Turks ruled in Central Asia. Architecture was a valuable intellectual endeavor, such as literature, music, and mathematics, and was appreciated by both the rulers and the public. As expected, in architecture progress was made in such an atmosphere, where important steps were taken in the development of theology and mysticism in the world of thought; astronomy, mathematics, optics and chemistry in positive sciences; music, miniature and calligraphy in art (Starr 2019, 402-503). It is also seen that Tulunids in Egypt, which is among the states founded by the Turks, decorated their very important buildings, especially Ibn Tolun Mosque, with very elegant geometric ornaments. The geometric motifs encountered in different geographies are quite similar to each other. This is an indication that they belonged to the same culture. On the other hand, the question arises whether the mathematicians who lived in that period also contributed to the formation of the crystallographic patterns encountered in the decoration of the architectural works, which are a reflection of the scientific atmosphere of the society.

### *A. Cooperation between Artists and Mathematicians*

The predominant use of geometric elements in Islamic art brings to mind the idea that the artists who draw these patterns are experts in geometry. It is a fact that architects and craftsmen who apply geometric ornaments are knowledgeable about geometry. Based on the similarity of geometric ornaments exhibited in Turkish-Islamic countries, it is thought that these patterns originate from a common doctrine. Some art historians cite a few assumptions about how these geometric patterns were created. One of these assumptions is that a small prototype of the geometric pattern is created on paper (there is no explanation about how it was created) and this pattern is applied to the surface with 1/1 scale (Mulâyim 1982, 52). Although we do not have enough data about the stages in which geometric patterns were created, the assumption is true to a certain extent. The last valid suggestion on this subject belongs to Alpay Özdural. In the article by Özdural on the relationship between mathematicians and artists in Islamic art, it is explained that the subject has a different background. According to Özdural, meetings, in which mathematicians and artists attended together and shared their experiences and knowledge with each other, were held. The first work in which these meetings are mentioned is the work titled *Kitâb fîmâ Yehtâcü ileyhi's-Sâni 'min A'mâli'l-Hendese* (*Geometric* *drawings needed by artists*) by Abu'l Wafa al-Buzjani. According to Abu'l Wafa, meetings with geometers and craftsmen were held in Baghdad. As far as it is understood, these meetings were a common practice in the Islamic world.

Another example was quoted by the famous mathematician and astronomer Omar Khayyam (1048-1131). In his untitled work, in which he describes the right triangle problem, which he solved with the help of a cubic equation, Khayyam states that the solution was completed in his mind thanks to a question asked at a meeting attended by craftsmen and mathematicians (Özdural 2000, 172). Likewise, Jamshīd al-Kāshī (d. 1429) states that he found the solution to the construction problem of the triangular leveling device during the construction of the Samarkand Observatory in a meeting with craftsmen, mathematicians and statesmen.

Apart from these examples, we learn from Cafer Efendi that the same practice was continued in the Ottoman period, and that meetings were held in Istanbul for a period of 20 years, corresponding to the period between the end of the 16th century and the beginning of the 17th century (Özdural 2000, 172). Cafer Efendi, again in his *Risâle-i Mimâriye*, states that apart from these meetings, the architects personally took geometry lessons from more senior architects. The most striking example in this regard is that Sedefkâr Mehmed Ağa (d. 1617) took geometry lessons from Great Architect Sinan (1490-1588) (Özyalvaç 2013, 1223).

### *IV. Conclusion*

When looking at an architectural building, the most striking and memorable aspect is its decorations. Turkish-Islamic artists used geometric ornaments in their works more often than ever before. In this article, we tried to show that the crystallographic patterns in Turkish-Islamic buildings are like the formation principles of crystal molecules that are the subject of crystallography. It is no coincidence that these patterns, which were produced in a period when symmetry groups or crystallography were not known, are considered as a kind of cultural element, especially in all geographies that Muslim Turks, such as the Karakhanids, Seljuks, and Anatolian Seljuks have reached. It is thought that the crystallographic patterns found in the decorations in the architectural buildings found in all these geographies are the pattern language of the Turks as a kind of cultural element.

Finally, we can say that considering the history of mathematics and crystallography, the crystallographic patterns used in architects are a manifestation of the knowledge of Turkish-Islamic civilization in many sciences, especially geometry. Mathematicians as well as artists of the period had a share in the creation of crystallographic patterns in architectural buildings. In other words, we can say that these patterns are the joint production of the mathematicians and artists of the period. Therefore, while investigating the level of knowledge of Turkish-Islamic civilization in the field of mathematics, plastic art elements such as architectural buildings should also be taken into consideration as well as written mathematical works.

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