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**Formation of Geometric Patterns in the Architectural Decoration:  
An Investigation on the Ilkhanids' Period**

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**Abstract**

The Ilkhanids made considerable changes in their ancestors' legacy after capturing Iran caused the creation of complex geometric patterns. Exhaustive research has been undertaken about the features of decoration of this era; however, the raised question is how this progress could have happened in spite of slaughtering or secluding the artists. The purpose of this study is to point out the role of the most effective factors in the formation of the decorations with complex geometric patterns during the Ilkhanids' era. This study implements a descriptive-historical approach with reference to written library sources. The analysis of the observations shows that Iranian rulers and scholars, especially Nasir al-Din Tosi, have played a key role in this development. This includes especially Nasir al-Din Tosi who is the composer of several fields including astronomy, ethics, history, logic, jurisprudence, mathematics, medicine, poetry, and philosophy as well he translated from the Greek geometry books. His success influenced Mongols. Eventually, he was appointed in several administration positions. His position influenced the promotion and progress of the geometry and math and their application in architecture and engineering. In fact, it can be said that the interaction of architects and mathematicians, which was Tosi's brainchild, brought about a turning point in the Ilkhanids' art and architecture, especially geometric decorations.

**Keywords:** Geometric Pattern, Decoration, Architecture, The Ilkhanids, Nasir al-Din Tosi, Islamic Art.

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## **1. Introduction**

Throughout history, the decorations of Iranian buildings were designed by three types of materials: brick, tile, plaster, or a mixture of all. The art of geometric pargeting and its booming, which is the subject of this study was originally used to cover the rough surfaces and their decoration, but in the fifth century AH it came out of its simple decorative form and changed into a complex, unique decorative cover. This art was used geographically vastly as a decorative element in architecture in the Ilkhanids' era and left unique works of art from this time. The distinguishing feature of Iranian pargeting in the Ilkhanids' era lies in its design and patterns (Pope, 2006). The patterns used in this art were the result of the creative, dynamic Iranian mind.

The Ilkhanids dynasty ruled Iran and the surrounding lands from about 653 to 736 AH (Egbal Ashtiani, Pirnia, & Javanmardi, 2010: 672) and the vassal governments of this dynasty such as the Mozafars ruled the country to the end of the 8<sup>th</sup> century AH. In fact, the art of this era is a mixture of the far east of central Asia and the arts of previous eras in Persian history. "Being inspired by these, the artists in this era tried to create original works and gradually changed these works to independent works" (Schrato & Grube, 1997: 388).

Despite slaughtering Iranian by Mongol governors during Ilkhanids' era, Iranian architecture resumed its booming path after a short interruption, and as a result, the complex geometric patterns appeared in architectural decorations. Consequently, in this era, the accompaniment of architecture and math in Iran, although witnessed in the construction and body of the buildings previously, was a display of the interaction of the architect and engineer, which undoubtedly brought about unique works.

## **2. Literature Review**

There have been numerous studies on Iranian architectural decorations. To name a few of them; using a descriptive-analytic design, Atefeh Shekofteh (2012), in "Visual Characteristics of the Decorations Index of the Architecture of the Ilkhanids Architecture", studied the features of the Ilkhanids' distinguished parget works and mentioned the particularity of these decorations over the previous eras.

Laleh Mehdi, Nejad Moghadam and Mostafa Goodarzi (2014), in "The altar and the patterns designed on it", studied the history and the application of different patterns including geometric patterns in the Seljuks' and the Ilkhanids' eras and compared these patterns in two Seljuks' and the Ilkhanids' buildings with regard to color, using positive and negative surfaces, and the broadness and complexity of the patterns.

Mehran Montasheri (2016), in "The study of the trend of development of brick decorations in Iranian architecture in mosques and the reason of underdevelopment in tile work from the post-Islam Iranian architecture to the Ilkhanids" in Jondi Shapoor quarterly, studied the developmental trend of

brick work in the architecture of mosques until the end of Ilkhanids with the aim of the introduction of decorations and identification of decorative forms and the reason of their continuity in the following years. He introduces Seljuks' era as the summit of brick work in Iran and then mentions the replacement of plaster in the Ilkhanids' era.

Bahareh Taghavi Nejad (2016), in "A study of Ilkhanids' geometric decorations of altars in Iran" in *Islamic Architecture Research Quarterly*, studied the use of geometric patterns in altars and their geometric patterns in strap work<sup>3</sup>. Her research revealed that the application of geometric patterns which are accompanied by arabesque and calligraphy are conspicuous in particular parts of the altars.

Yahya Abdullahin and Mohamed Rashid Bin Embi (2013) from Malaysia, in "Evolution of Islamic geometric patterns" in *Frontiers of Architectural Research*, came to this conclusion that the circle and its center are the points from which all Islamic geometric patterns start. This is the symbol of oneness of God (monotheism) in Islam.

As it can be seen, all of the previous studies on geometric architectural decorations and their developmental process were conducted meticulously and carefully. However, to the best knowledge of the author, the cause of the formation and prevalence of geometric patterns during the Ilkhanids' era have not been studied so far. Nasir al-Din Tosi and his influence in all aspects even in arts have been emphasized. For instance, Dr. Hasan Bolkhari studied the Islamic art in the Ilkhanids' era and the two elements of imagination and weight as the main constituent factors of art from Nasir's point of view which was presented in a congress held at the honor of at Jawaharlal Nehru University in Delhi in 2004. Therefore, in order to answer this question, the researcher studied the social conditions of that era to find the most determining and influential factor in the formation of geometric shapes taking advantage of historical-descriptive method and relying on the written sources.

It is worth mentioning that the social-political conditions of the Mongols' ruling era have been studied frequently in all of which the prominent role of

Nasir al-Din Tosi's collection of poetry and its relation to art and architecture were studied by researchers such as Mohammad Shole Saadi and Omid Yazdanloo in 2015. *Master of Human*, which is a collection of eight articles presented at the conference held at the honor of Nasir Tosi in 2013, depicts the conditions of Iran in Ilkhanids' era and Nasiri's influence on the social, political, and cultural conditions.

### **3. Methodology**

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<sup>3</sup> - Strap work is a delicate and rich technique that is made with bricks cut and scabbard. There is an example of strap work in this period in the minaret of Jameh Mosque of Saveh and Damghan Jami Mosque

As it was mentioned above, the salient point in the process of Iranian architectural decorations is the changing of Seljuks' simple geometric brick work decorations into the Ilkhanids' complex parget. Seljuk's geometric decorations with regard to brick work geometry were not difficult, but building complex geometries using plaster is contemplative and requires a specific condition. As a result, the researcher discusses the formation process of decorations in different eras until the Ilkhanids' in order to understand the relationship between the decorative geometric plaster patterns and the social conditions. In the next step she will find the relation between these two issues and come to a conclusion by mentioning the dominant conditions and the influential people in the society.

#### **4. Decorations of the Ilkhanids' Era**

Decorations have always held a special place in architecture and have been an inseparable part of the buildings in Iran. This art has provided a broad link between other sciences and architecture both in interior spaces and exterior surfaces. History has shown that these innovations have been spurred by people's relations, interconnections of the cultures and different sciences in the social milieu of that specific time. Parget with geometric patterns found a special place and importance in architectural decorations of the Ilkhanids' era. As this research investigates the geometric plaster patterns in the Ilkhanids' era and its changeability by the social situations, the geometric patterns of this era will be discussed after a brief introduction to the developmental process of the decorations until the Ilkhanids' era.

##### **4.1. The historical background of the decorations**

With the beginning of the historical period and the growth of human civilizations, the issue of decoration was also considered, followed by the correlation of decorations with architecture. In Iranian art, especially after Achaemenes, using decorative motifs in architecture was developed. This trend was developed in Islamic era so much that the symbolic figures alongside architectural components and potency not only were effective but also surpassed in conveying concepts. This art of decorations creates four different patterns and motifs with three main elements of brick, plaster and tile. Changing the natural shapes to abstract formulas, this art gives them an esoteric feature. These four patterns include plant patterns, geometric patterns, symbolic patterns and inscriptions. In the following section, the historical background of decorations of pre and post Islam until the Ilkhanids' era will be discussed.

##### **4.1.1. The Decorations before Islam**

The Decorations before Islam were mainly in the form of brick work. Of course, there are some reports regarding the simple samples of plaster that can be seen in historical buildings" (Naeima, 1997: 17). The decorative element of brick history in Pre-Islam Iran dates back to Ziggurat Chogazanbil in Ilami era (Figure 1). "In addition to brick inscriptions, an example of which is seen in the Figure, in this building, there is a façade with red baked brick and green and blue brick" (Golabchi, 2013: 341).

After the Ilams and taking power of the Achaemenids, they also used brick with plant patterns and mythical animals in the decoration of their buildings. “The remained proof from this era is Apadana



Figure 2. Shush Enamel Bricks

Palace in Shush (Figure 2), which includes a row of lions on the brick” (Wendenberg, 2000: 80).



Figure 1. Bricks Inscribed in Choghazanbil

### 2.1.2. The Decorations after Islam

In the early centuries of the arrival of Islam (1<sup>st</sup>-2<sup>nd</sup> century AH), the buildings were built according to past patterns; however, the tendency for height reduced partly due to the promotions of simplicity and populism inspired by Islamic doctrines, so brickwork was not observed in the decorations of the buildings any longer. Consequently, brick was merely used as the construction material; the two buildings of Damgan Tarikkhane and Fahraje Yazd mosques are examples of this brickwork.

With the beginning of the 3<sup>rd</sup> century AH, decorations became so commonplace in the buildings that Ale Buye era is considered as the beginning of using the art of brickwork in façade especially in mosques and its summit was observed in Seljuks’ era.

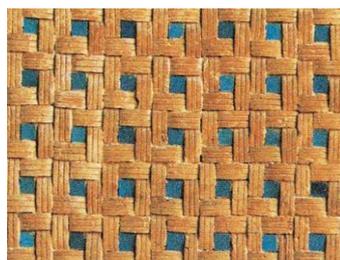


Figure 3. An example of primitive geometric patterns in the form of a mat in the Jorjir Mosque

From the 3<sup>rd</sup> to 5<sup>th</sup> century AH, the first examples of geometric patterns in brickwork decorations became commonplace and reached their summit (Figure 3). The portal of the Jurjir Mosque is one of the examples of exquisite brickwork in the second half of the 4<sup>th</sup> century. This portal, related to Ale Buye, is the representative of one of the most distinguishing features of this era, that is, the integration of architecture and engineering. Its brick decorations include abstract patterns which have been designed in geometric faulting labyrinthine shapes (Figure 4).



Figure 4. Geometric patterns of the eight ridge rhombus in the Jurjir Mosque

The 5<sup>th</sup> and 6<sup>th</sup> century AH, contemporaneous with Seljuk s' ruling era, are some of the most important times regarding the Islamic art. During this era, complex brickwork with a variety of patterns was offered. The noteworthy point during this era is the function of brickwork that was not merely used decoratively. For instance, the decorative brickwork of the arches of Isfahan was also used for the purpose of bearing the pressure. In other words, decorations were used for the purpose of decoration and construction simultaneously (Figure 5).

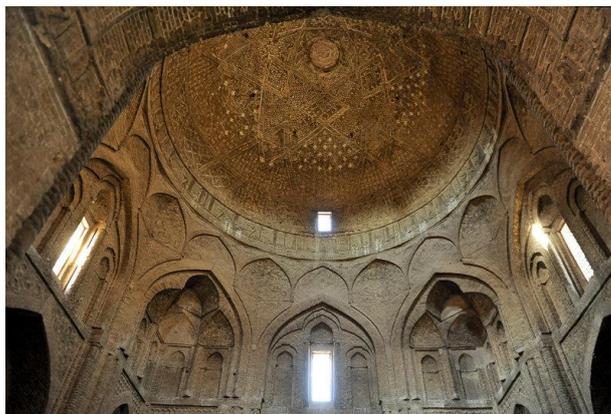


Figure 5. Brick decoration in the form of a structure of the dome of Taj-ol-Molk Imam Mosque of Isfahan

The brick decorations in Seljuks' buildings have been used variously and widely. This era is the summit of brickwork in Iran, but in Khwarazmian's era, especially the Ilkhanids, brickwork was not used as widely and tile work replaced brickwork. The changes occurring in architectural decorations in

Seljuks' era caused the evolution of patterns and decorative techniques in the following eras. In Khwarazmshahian' era, Seljuks' bricks with patterns (patterned potteries), by which they decorated many surfaces, became commonplace (Figure 6). Various strap work patterns which were used during Seljuks' era to decorate the exterior surfaces, became the source of inspiration for hybrid brick, tile and brick, and parget (Figure 7).

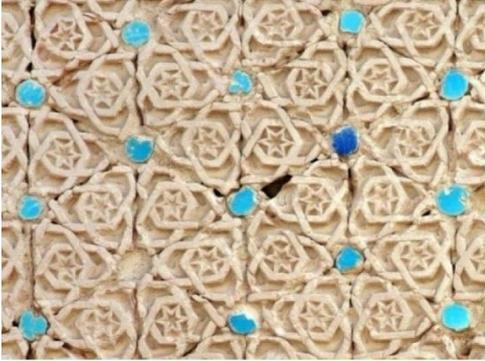


Figure 6. Geometric patterns of brick and tile in the Freimud Mosque



Figure 7. The Mohammed Inscription Brick and the tile bars to create a geometric node plan at Soltaniyeh Dome

Transoxiana (Mawara an-nahr) and Khorasan regions were under the control of Khwarazmshahian between 628-492 AH. The architecture in this era was similar to Seljuks but due to the Mongols' invasions and the destructions, a few of the works, which are unique, are remained. The Malek Grand Mosque of Zuzen, Faryumed, Gonabad, which are known for their brickwork decorations, remained from this era. In this era, the art of brickwork was similar to the previous era and the integration of brick with tile, which had started from the previous era, was used widely. The tile with three colors of sapphire, indigo and white, which was a considerable step in the invention of mosaic tile work, came into use and reached its summit in Teymuri era. Also, in this era, the best use of geometric patterns in brickwork decorations in the mosques of Khorasan including the west Eivan of Zuzen Grand Mosque can be observed (Figure 8). One of the examples of the decorations of this era is Gonabad Grand Mosque, which was decorated by geometric patterns in Islamic architecture (Figure 9).



Figure 8. Geometric patterns in Zozan Mosque

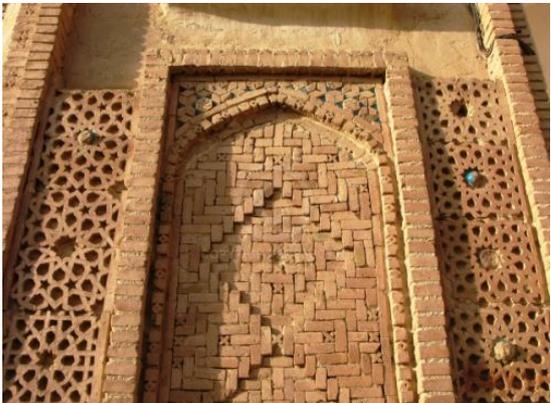


Figure 9. Geometric patterns in the Gonabad Mosque

The Ilkhanids' government, which ruled the vast area of Iran, Asia Minor and sometimes Syria over one and a half centuries, started with Hulagu Khan's (653 AH/ 1255 AD) taking the power and lasted until the death of Abu Saeed Bahador Khan (736 AH/ 1336 AD) whereas the unstable government and the ruling of few princess and Changiz Khan's descendants and their princes lasted in some parts of Iran and Iraq until the late 8<sup>th</sup> century (Bayani, 1988: 167). Due to Mongols' nomadic style of life and conquering territories, construction work encountered a temporary recession; however, they attempted to build up the country and construct countless buildings hastily by taking advantage of resourceful ministers and relying on their art loving and encouraging spirit; therefore, they used their best contemporary artists and experts' abilities. This effort, led to the creation of some of the most important buildings, which spread widely throughout contemporary Iran. The boom in various constructions during the Ilkhanids' ruling required a decoration more than before; therefore, tile work in addition to brickwork and varieties of parget became the distinguishing features of this era. Although, the use of plaster was commonplace in the previous eras, the decorations of parget with regard to the design and technique of the pattern is contemplative. Different kinds of parget include Kofi calligraphy inscriptions, Sols, Naskh, plant and geometric patterns which were used in different parts of the building such as the wall surfaces, vault, altar, the edges and so on.

Overall, during Kharazmshahian and Ilkhanids' era, they made the best of tile alongside brick, and created beautiful patterns in some decorations with the combination of bricks or patterned pottery and tile in the shape of carving or block. In fact, Ilkhanids' era is known for the summit of the combination of brick and tile technique.

#### **4.2. Gradual evolution of geometric patterns of motifs in decorations**

As it was previously mentioned, the application of geometry in art, especially architecture and decorations related to it were commonplace during the eras before Islam, but the transition to Islamic era was so effective in the development of geometry that most of the complex geometric rules in the Islamic Iranian civilization were known and displayed in art and architecture. The geometric patterns from the second half of the 4<sup>th</sup> century with beautiful plant patterns had been used to decorate the historical buildings. Until the Ilkhanids' era, geometric patterns in brick decorations were more widely used than the other decorations, which was because of the corners and right angles of the brick to design geometric patterns. The geometric decorations lasted to the Ilkhanids' era and reached their summit in the field of plaster with plant and geometric patterns and inscriptions. During that era, the artistic architect did not skip any observable space without subtle parget.

One of the fields of development of geometry pertains to the use of rhythm in complex geometric designs. The creation of separate motifs to design geometric patterns was another field of development, which led to the creation of motifs of the strap work. The simultaneous use of geometric patterns with other decorative elements along the main lines include the motifs leading to the center, which create a polygon shape known as rosette which attract all attention to the design. The goal of this art was the movement and combination of components to achieve unity in multiplicity. The important point in drawing these decorations was that for centuries the compass and ruler were the

only tools to create polygons and the required angles; therefore, the drawing of polygon from a circle and the divisions of its perimeter was one of the most common ways of designing polygons. Sometimes in these patterns in the space among polygons, stellar shapes were created, which caused dynamicity in a negative atmosphere. Also, two or polygons were commonly drawn within each other in complex geometric patterns and created complex shapes. These patterns were extended to cover a vast surface of the building and were formed of compound repeated patterns (Figures 10-11). The template of geometric decorations and their analysis in the Ilkhanids' era will be discussed in the following sections.

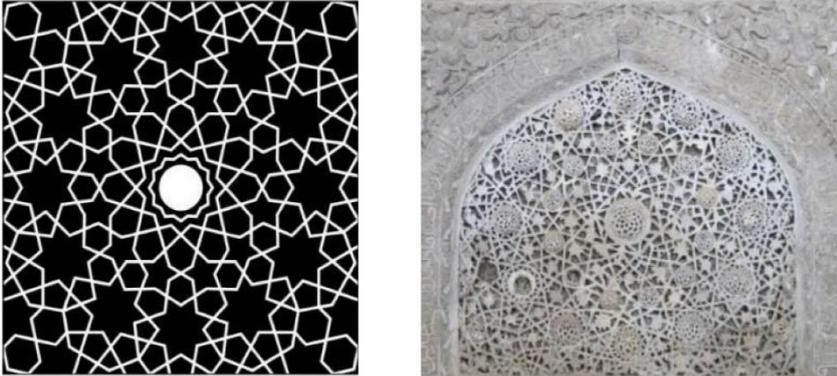


Figure 10 and 11. The linear design of the node of eight and twelve in the mosque altar of Orumiyyeh Mosque

4.2.1. The geometric template of motifs before the Ilkhanids' era

Most of the decorations in the Ilkhanids' era were brickwork. Robot Sharaf Caravanserai and Golpaigan Grand Mosque, which include a variety of brickwork patterns and settings, are some of the typical examples of these impressive buildings. These two buildings, more than variety of the brickwork, have taken the advantage of the combination of brick and plaster extremely beautifully and skillfully (Figures 12). The geometric template of various brick layout which were used in the eras before the Ilkhanids were as presented in Figure 13.



Figure 12. brick in Robot Sharaf

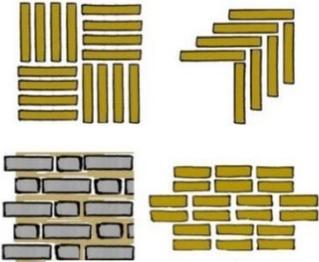


Figure 13. brick Layout

Flower Design motifs during this era create a variety of patterns in different combinations. These motifs are divided into two kinds: simple and complex (Figure 14).

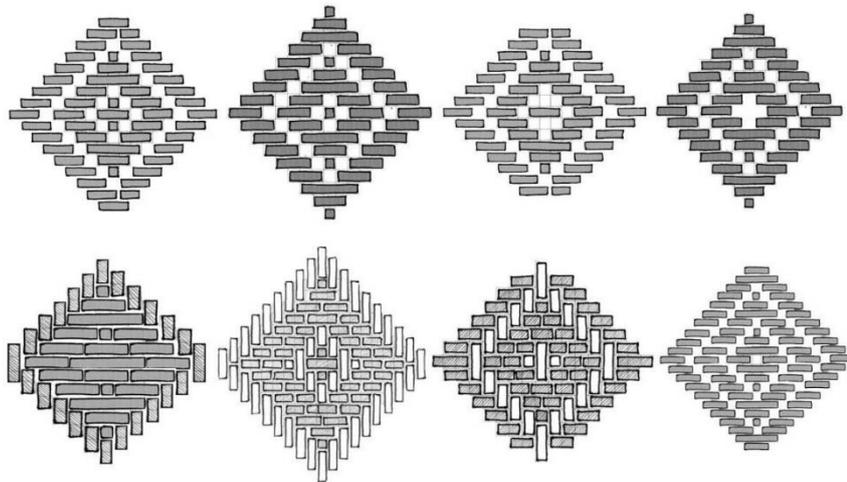


Figure 14. The Plan of the Prehistoric Period of Ilkhani in the Decoration

The most extensive use of zigzag brickwork is in the building of vaults and fountain vaults, but wicker motifs were of rare use in the Ilkhanids' era. The other templates of geometric decorative brickwork such as Khofteraste (corbel), Fakhr and Modin (cellular) brickwork, strap work, the combination of brick and plaster in different kinds of inscription together with floral parquet were commonplace in decorating the buildings of pre-Ilkhanids era (Figure 15).



Figure 15. Nodding the mosque in Damghan Jamea Mosque

In this period, for the first time, besides bricks, the prefabricated bricks and sometimes chips from gypsum and tiles were used. In other words, the integration of different kinds of materials in a motif, which was also used in the other eras, is one of the inventions of this era. It should be noted that in addition to these techniques, the decorative margins with lozenge, circle and S-shaped bricks are

observed adjacent to each other in the dome of the Golpaigan Grand Mosque, the Grand Mosque of Isfahan and the building of the Barsian Mosque (Figure 16).

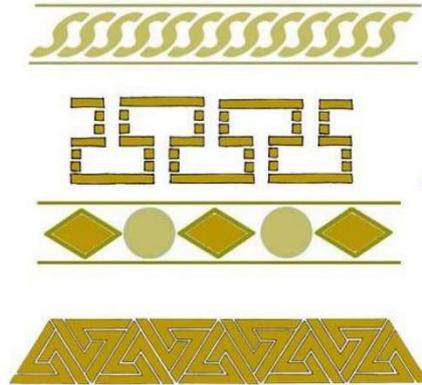


Figure 16. Common Brick margins in Brickwork Decorations in the Prehistoric Period of Ilkhani

#### 4.2.2. Geometric pattern of motifs during the Ilkhanids' era

After reaching the elements of Iranian architecture, The Ilkhanids tried to decorate the buildings with brick and plaster. During this period, the brick roof entered a stage of great development of the technique and performance, and the impressive and well-formed geometric domes like Soltaniyeh, and the brickwork as decoration with the same style of the previous period start declining. In contrast to the decline in brickwork and the less willingness to use this type of decoration, artists turned to parget, and this art was recognized as the dominant decoration of the period; in fact, geometric patterns which were used in the simple form in Iranian architecture in the form of decorative nodes from the 3<sup>rd</sup>-4<sup>th</sup> AH century, became common place during the pre-Ilkhanids (Seljuk) era and left unique samples, and in the period of the Ilkhanids' era, the motifs of the nodes appeared richer and more complex in the field of plaster in comparison to past as a result of geometric designs. Thus, the art of the application of geometric designs in this period became distinct and special. Before the Ilkhanids, the first example of the geometric pattern of plaster is seen in the altar of Nayin mosque related to the second half of the 4<sup>th</sup> century AH (Figure 17), on outer margin of it (Sajadi, 1996: 233-235).



Figure 17. The altar of Nayin's Grand

In the Ilkhanids' era, the application of repetition of a base pattern in geometric decorations is a feature of geometric patterns, which, at first, was created with less variety and then with more motifs, creating richer and more complex nodes.

According to the research by Jamaloddin and Kakhki (2017), which was conducted on the decorations of the Ilkhanids' period, the most applications of geometric designs belong to the strap work constructed on the basis of the number 6 and 8, such as the strap work of the sixth and the and Shams (like the sun) (Figures 18, 19, 20, 21), Octagonal nodes (Figure 22, 23), eight and twelve, as well as stellar work.

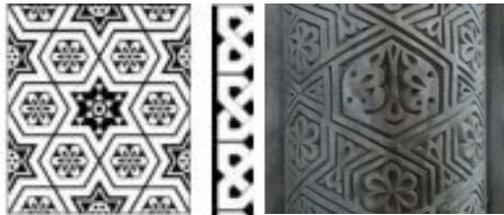


Figure 18. The sixth and Shams (Like the sun) node in the altar of Imam Zadeh Fazl and Yahya Mahallat

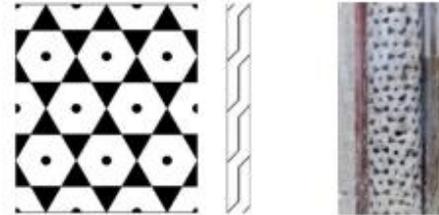


Figure 19. The sixth and Shams (Like the sun) node in the altar of Imam Zadeh Fazl and Yahya Mahallat



Figure 20. The sixth and shams (Like the sun) node of the altar of the monastery Bayazid Bastami

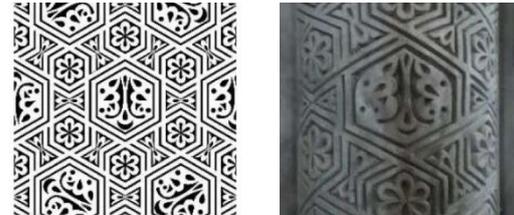


Figure 21. The Linear Pattern of the Sixth at the altar of the Bayazid Bastami Mosque

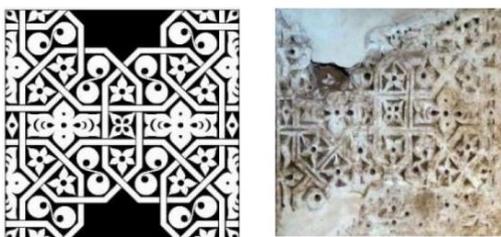


Figure 22. Linear pattern of Octagonal nodes in the Bayazid Bastami altar

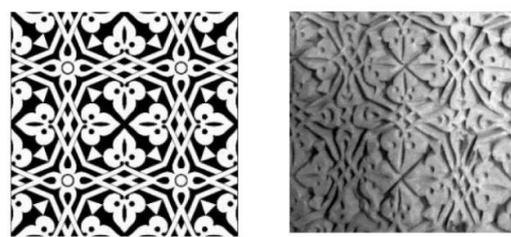


Figure 23. Linear pattern of the eighth & fourth nodes in the Bayazid Bastami altar

However, with the advent of philosophers and cosmologists such as Nasir al-Din Tosi who made an important contribution to the nature of numbers and their relation to nature and geometry (Critchlow, 1989), the patterns and symbols of mystical Tetrarchy's were merged with the traditional geometry of patterns. As a result, abstract geometries based on the Tetrarchy's symbol and patterns of 12-point stars were created. Another number associated with mysticism and cosmology was seven that was linked to seven heavens in Islamic perspective. This number is used in the construction and production of a 14-point star, which is a symbol of fourteen holy people, especially for Shia Muslims. Therefore, despite the difficulties in making heptagons, Muslim designers created an approximate

approach to drawing heptagons (7-point stars) and other geometric patterns. The first 10-point geometric pattern also dates back to the 11<sup>th</sup> century; the number 10 is not only related to the Tetrarchy's symbol, but also is twice as big as the pentagon. The 9-point geometric pattern is another example of the influence of cosmological thoughts on geometric decorations, especially in the 11<sup>th</sup> century. Approximate methods for drawing 7 and 9 polygons and hidden stars, led Muslim architects and artists to use practical methods to draw polygons of 11 and 13 angles and their stars. Some of the first samples of these decorations are in the Barsyan Mosque (1098 AD) in Iran. In the late 12<sup>th</sup> century, the method of making polygon, stars and patterns based on simple or single-layer circles were recognized throughout the Muslim realm. This development was followed by the introduction of complex and dynamic features of the grids of the circle. More simple types with a subgroup, with a central grid point and a central point, but at different scales developed (Abdullahin & Bin Embi, 2013).

Thus the patterns of six and eight points derived from divisions of the circle perimeter, which was the dominant pattern of decorations in the pre-Ilkhanids (Seljuk) period, and which is visible under the title of the first art movement with patterns of combination of 6 and 8 points in the Kharqan Tower of Qazvin, developed during the Ilkhanids era. The second movement of art continued to change with the use of traditional patterns of the past and the creation of new, more abstract and more complex patterns of 5-7-9-11 points in the unique decorative composition. Some of the examples can be seen below (Figure 24-27).

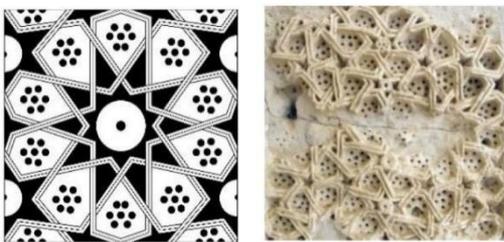


Figure 24. Six and ten nodes in the altar of the Great Mosque of Abarkuh



Figure 25. Eight and five nodes in the altar of the mosque of Abrakuh



Figure 26. Eight knot and the drum in the altar of the Imamzadeh Rabie Khatun, Ashtarjan, Isfahan



Figure 27. Five and twelve nodes in the Imamzadeh Rabie Khatun, Ashtarjan, Isfahan

## **5.The Ilkhanids' decorations and Nasir al-Din Tosi**

As stated, extensive progress was made in the field of geometric patterns of architectural parget during the ruling period of the Ilkhanids Mongols. To study the reason of this progress, the context and social conditions of this era should be investigated.

During this period, the violence of the Mongols caused lethargy and passivity in the society, which was in opposition to the spirit of the Persians and put them in adverse circumstances. Nevertheless, the state sought to integrate Iranian elements in order to gain legitimacy. In fact, the convergence and interaction were a policy of the state to moderate the ruling system to deal with society and culture, and align with indigenous elements, in which the role of the culture and thoughtful stratum of Iranian society was very effective. Among these people, the author of this article seeks to share and review the role of the well-known and sharply distinguished personality of Nasir al-Din Tosi as a realistic and successful philosopher in the tranquility of the society and the growth of educational centers of the country, including mathematics and architecture.

### **5.1. Socio-cultural conditions of the Ilkhanids' era**

The Mongol invasion led by Genghis Khan in 616 AH to Iran and the massacre and destruction of many of the cities of Mawara an-Nahr and Khorasan were the most devastating catastrophes that the Iranian people suffered throughout history; until forty years after the first Mongol invasion, there was no integrated government in Iran.

In a number of regions, local governments ruled out arbitrarily, and insecurity, chaos and turmoil in affairs aggravated the social situation of the people until finally at the Mongol invasion under the leadership of Hulagu Khan, Ismailis' power, who mostly settled above the mountains and in the fortresses, and were terrified by ordinary people, as well as the Abbasid caliphate fell, and was followed by the establishment of Ilkhanids government in Iran. Consequently, the formation of the Mongolian rule was accompanied by a military overthrow of religious and political institutions, as well as by the bloodshed and killing of the people and the destruction of the cities. From the very beginning, the Iranian people felt hatred towards the Mongols' government, and as no formations and unity were formed among them, malicious feelings were accumulated in the hearts of the people. The continuation of these conditions affected the life of the Islamic government; therefore, they sought solutions to improve their socio-cultural status (Reza zadeh, 2013).

### **5.2. Interacting with Rulers to Survive the Mongols' Ruling Crisis**

In a situation where the Iranian society, under the Mongols' rule, was faced with a huge crisis, it was natural for a well-educated and efficient urban community to seek a way out of this crisis by relying on their past experiences. The first attempt by the cultured people of Iran, their consensus on the formation of the Ilkhanids' government as an alternative solution, was to fill the existing political vacuum with the aim of restoring order and relative security.

Understanding the various rounds of the assault of the nomads of Central Asia, the Iranian society had convinced the cultured and thoughtful community that peaceful co-existence with

Mongols was the best way to overcome the existing crisis, instability and insecurity. Therefore, by adopting a policy of close proximity to the Mongols and attempting to push the confrontation of society and government towards interconnectivity, they helped to consolidate the foundations of the power of the Ilkhanids. Of course, the Iranian society did not approve the illegitimate Ilkhanids' rule, which was bizarre to Iranian civil code; but it had the only way to accept and co-operate with it to attract the domineering people. The acceptance and engagement provided the opportunity to control the Mongols' behavior and to modify their political and economic approaches as the main strategic and effective discourse and made the Mogul period (654,616 AH) an effective era.

### **5.3. Nasir al-Din Tosi, the Philosopher and Minister of the Ilkhanids**

Muhammad Ibn Hasan Maki Abu Ja'far, known as Nasir al-Din Tosi (born in 597 AH in Tus, d. 672 AH in Kazemein), was one of the great scholars of astronomy and wisdom of Iran in the seventh century and one of the ministers of the Ilkhanids period. He was a philosopher, orator, astrologer, mathematician, and anthropologist from Iran, who was known as Khajeh, Professor Al-Bashir, Mohaqiq Tosi (Tosi the researcher) and Nasir al-Din. After taking religious and rational science lessons from his father and logic and natural wisdom from his uncle, was able to complete his education in Neyshabur, where he became famous as a renowned scholar. He authored several manuscripts on mathematical sciences, astronomy, logic, natural sciences and divine wisdom.

He lived in one of the most critical periods of the history of Iran and Islam. This tumultuous era, which was dominant shortly before the Mongol invasion in the eastern Islamic tribe, encountered a widespread crisis with the Mongol invasion, and forced to join the Ismailis and achieve knowledge about the political, ideological, and thinking pattern of Nizari along with the experiences of the Khwarazmshahian. He took advantage of the security of the Nizaris' castles, both in Quhistan and in Alamut, to gain maximum knowledge, do research and write books; he became an expert in the various fields of sciences. His remarkable reputation, in the obedience of the last Imam Nizari of Hulagu, took him to the Mongol's heart. From the conquest of Alamut in 654 AH / 1256 AD to the conquest of Baghdad in 656 AH / 1258 AD, this great and prudent man became so influential that, in addition to advising on serious affairs, he was given the mission to set up the Maraghe Observatory, as the largest Ilkhanids Cultural Research Center and a site dedicated to gathering scholars, books and the intellectual achievements of the Islamic world, establish order in tax affairs and take the custody of the endowments of the countries with the conditions that he himself had set.

Nasir al-Din Tosi, in spite of the active cooperation with the Ilkhanids and catching their undivided attention to himself and his family, devoted most of his time to the cultural activities. Considering the number of his works and variety of research and compilations, he was the greatest and most prominent thinker of not only the Middle Ages, but also throughout the Islamic era. Tosi, in particular, had a global vision and insight, and his tolerance of beliefs and actions gave him a strong personality; he had such a political, economic, cultural and social power that contributed significantly to the evolution of the conditions of society and government.

Nasir was the philosopher of dialogue and interaction; behind his conversation and his conduct, there was a profound discourse whose foundations he used to revive the optimal conditions of the Iranian community and to repair and correct it. Although at the above-mentioned cultural challenge, Tosi was not alone, and in the same way all the great Muslims, including government officials, poets, scholars, jurists, mystics, etc., came to the forefront of the support of the Jovini family, seriously confronting the Mongols to attract them, Nasir al-Din Tosi held a different from the others (Razavi, 2013). Nasir al-Din Tosi was involved in various activities during this period, including the following:

- The transformation of the Shiite's secret and negative struggles based on the principle of secrecy (taqiyah) into open struggles and facilitating the spread of Shi'ism, which later became known as the official religion of the country.

- Revival of the endowment of the country, which was the ancient Islamic tradition and one of the economic pillars.

- Creating a new era in the knowledge of astronomy and mathematics.

- Establishing a new style in architecture and a variety of arts.

- Making a major contribution to preserving past culture by collecting books and improving the position of the intellectuals of that era (Bayani, 1992: 353).

In the following section, we examine the impacts of Nasir al-Din on mathematical sciences and geometry in order to discover the process of the formation of complex geometric decorations in this period.

### **5.3.1. Nasir al-Din Tosi and mathematics**

A large part of Nasir's writings was on mathematics and astronomy about which 19 writings were left. In reviewing the relevant books, in order to achieve the purpose of the study, the book of the *Writing of Principles of Euclid* was found to be an appropriate guide. This book was a reproduction of *Principles of Euclid* in Arabic and was the most important book on mathematics in this period. The book *Principles* by Euclid (the most famous mathematician of the Ancient age, known as the Father of Geometry) was the first and most famous work in geometry, in which Greek mathematics was scientifically arranged up to 300 BC and is still the basis for the teaching of elementary geometry today. Hajjaj Ben Youssef Matri Kofi was the first person to translate this book from Greek to Arabic during the time of Harun al-Rasheed. The scholars of the Islamic world also worked hard to eliminate the shortcomings and make use of this book (the principles of Euclid) as a textbook; they provided a number of extract and scripts, such as Mohiyeddin Maghrebi and Asireddin Abtari, but the *Writing of the Principles of Euclid*, the ecclesiastical allegory, overshadowed all of them and became one of the most read texts in this period.

The book entries (*Euclid's Principles*) are briefly outlined in 15 chapters: First article: Preliminary definition, Principles and general principles of geometry, second article: Conversion of area and geometric algebras, third article: Including theorems on circles, chords and measuring their angles,

article four: Geometric drawings (Drawing regular peripheral and encircled polygons), article five: Proportion theory, article six: The application of equivalence theory in flat geometry, articles seventh to ninth: All in the preliminary number theory (including machine-tool proportions, geometric manifestations and first numbers), article ten: irrational numbers using their drawings as incompatible lines, articles 11-13: Spatial geometry, 14-15: Supplementary papers in the form of supplements on flat and spatial geometry.

It should be noted that Tosi, before the *Writing of Euclid*, created *the Writing of Almagest of Ptolemy*, in addition to that, he gathered a collection of books by mathematicians, "Archimedes" and "Apollonius" and several other Greek scholars. All of which were like a bridge for a math learner that led him to Nasir al-Din's studies. However, *Euclid's Writing*, played a large role in the education system of that time, and its prevalence over other books was due to the position of Nasir, who presided over the most important educational institution of his time in astronomy and mathematics, that is, the Mirage Observatory.

## **6. The Ilkhanids' decorations and analysis of the main factors of geometric changes of motifs**

As mentioned previously, the geometric patterns of the 6 and 8-point patterns were the most comprehensive Islamic decorations for centuries. These patterns with circle perimeter grid were introduced in the Ibn Toulon Mosque. Grids were used as constructive foundations for the simplest regular and semi-regular tiles with equilateral triangles, squares, hexagons and octagons. During this era, apart from the distinction between decorations, the simplicity in the construction of these geometric patterns led architects to use such decorations in almost all elements of construction; in Iran and Central Asia, of course, the abstract complexity and complexity of geometric patterns, limited their application to the viable elements (the walls of the kiblah and windows). The stable government in the Ilkhanids' era and the domination of the efficiency of ministers, encouraged the architects of this era to design beautiful and precise decorations with unique complexity. Thus, the transformation in simple geometric shapes made of a circle and a set of tangents with the same radius (Critchlow, 1976), began to appear in the Bastam Mosque.

In this section, according to the purpose of the research, two examples of decorative motifs in the period of the Ilkhanids and before the Ilkhanids, both of which are considered as their turning point, are discussed. The first example, Alavian dome, is related to the pre-Ilkhanids' era and the second example, the Bastam Mosque, belongs to the period of the Ilkhanids.

### **6.1 Geometric patterns in the Alavian domes (Seljuk era)**

This dome is one of the late Seljuk's monuments in the sixth century, which was originally built by the Alavian dynasty as a mosque, and later converted into a tomb in the basement by creating a cellar in the basement. The reason for naming this dome as Alavian dome is that it had a dome in the past, but over time, this dome collapsed. Other reasons were the people's interest in Sadat and love of Imam Ali and the burial of two of the Alavian families in this building. The rectangular altar of the Alavian dome has two overlapping soffits, constructed on a crescent-shaped pillar, and these soffits have four pillars with vase shaped tops. Professor Pope has described the paraging of the altar as an

exaggeration; he describes that considering the skill of the creation, beauty of the pattern and the beauty of the details, this interior pargetting equals the most exquisite pattern of eastern carpet. The margin on the altar of this dome is in the Kofi script and verses of the Quran are written on the field of flowers and arabesque shrubs. On the pillars of the altar are geometric pargetted patterns, and on the most of the flowers inside the soffits and spandrels and are covered with octagonal, stellar. Geometric patterns are seen with straight and broken lines within the frames of pargeting.

A cross motif with combinations of lotus flowers or palmate (a kind of plant) in geometric compounds is often used separately and individually. Sometimes curved and circular patterns and sometimes ring-shaped chains or lotus like flowers appear in different sizes in the center of the pattern of the cross in the pargetted background.

The pargetted margin around the altar is decorated with geometric and the center with simple, irregular and somewhat rough plan patterns. Although there are fine geometric and arabesque wicker like patterns inside the leaves, most of the decoration consists of floral designs and adjacency with geometric patterns is less visible in this altar (Figure 28).

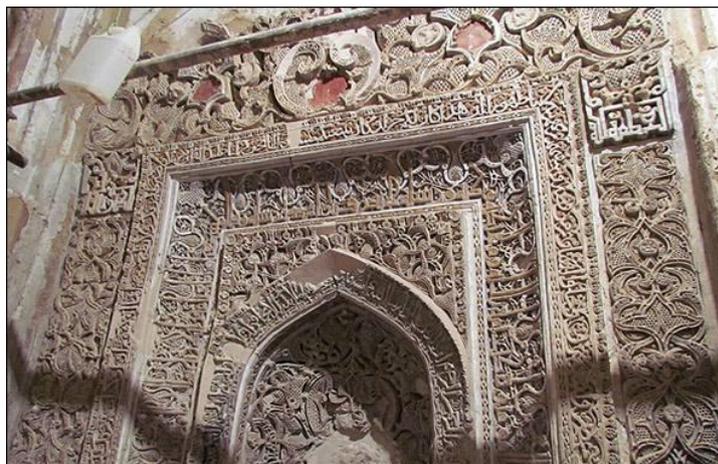


Figure 28. Alavian dome altar

## 6.2. Geometric Drawings in the Bastami Mosque (The Ilkhanids' era)

The building dates back to the Ilkhanids' era, in the city of Bastam, Semnan Province, and is located about one hundred meters far from the south of Bayazid Bastami complex. The mosque building consists of an entrance portal, a central courtyard and three seraglios on the eastern, northern, and southern sides, and unlike most Seljuk and the Ilkhanids' mosques, lacks yard, porch and minaret. The original mosque was square with open space and without a roof, but today the mosque is covered (Figure 29).



Figure 29. Altar of Grand Mosque of Bastam

The altar of the mosque is decorated with pargetting, including arabesque, geometric, and inscription. This altar has three fringes in the Kofi script and Naskh which is decorated with arabesque patterns. Arabesque has filled many positive and negative spaces of the inscriptions so much as if every little branch and leaf of it is arranged in a very precise and organized order in the complex. Geometric plant designs and complex designs in this mosque are similar to the Alavian dome. Geometric designs in the inner space and columns of the altar of the Grand Mosque of Bastam with volute shape have been designed in a systematic and orderly form with the aim of creating a kind of sense of unity. These paintings sometimes appear alone and sometimes alongside inscriptions and plant designs, including the arabesque. In the Figures below, a number of geometric nodes of this mosque are depicted (Figures 30-33).

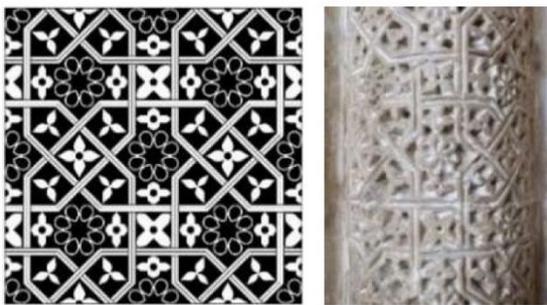


Figure 30. The six node in the altar of the Bayazid Bastami mosque

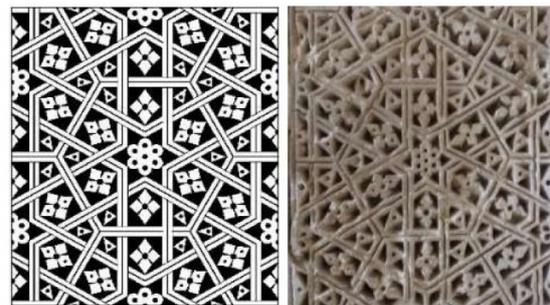
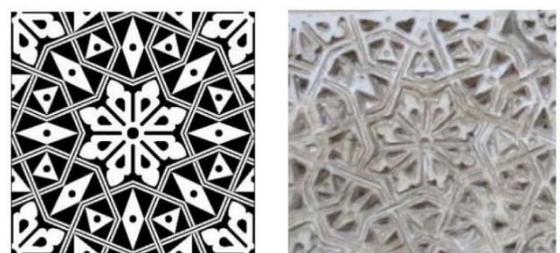


Figure 31. The six node in the altar of the Bayazid Bastami mosque



Figure 32. The linear node in altar of the Bayazid Bastami mosque



348 Figure 33. The eight node in the altar of the Bayazid Bastami mosque

### **6.3. Comparing and aggregating the factors**

It can be concluded that in the Alavian dome, elaborate designs and decorations were more detailed and more prevalent than pre-Ilkhanids patterns. In both altars, geometric designs are most seen in the margins and plant designs in the center, and the inscriptions are accompanied by arabesque designs. In the altar of the Dome of the Alavian, the positive atmosphere has received more attention than the negative space and the patterns are richer and coarser; coarse and fine motifs are seen together, but in the altar of the Bastam Mosque, the motifs are more complex, more complete, lighter, more regular and proportionate. Also, the positive and negative spaces have equal value and also have more order and evolution and a more regular scale. This evolution is seen in plant and geometric lines and patterns in comparison of the two altars. In the great capital of the Bastam Mosque, plant designs are similar to animal designs, and in the Alavian dome, motifs similar to animal motifs, which are simpler and more primitive than the Mosque of Bastam, can be seen at the end of the branches and leaves of the tree of life in the main altar.

### **7. Discussion and Conclusion**

As it was mentioned in the previous sections, the decorations in Iran's architecture have a long history of brick, plaster and tile. The historical background of brickwork decorations dates back to before Islam and the Elamite era, to Choghaznabil ziggurat. After the Elamites, the Achaemenes used their glazed bricks with plant designs and mythological animals in their important buildings. However, with the advent of Islam in the first centuries AD, and by promoting of the simplicity of the teachings of the Islamic religion, brickwork was used only in structural form in the buildings. Gradually, with the beginning of the third century and the rule of Ale Buye, brickwork decorations were re-enacted in the buildings and reached their peak during the pre-Ilkhanids (Seljuk) period. One of the remarkable points in the Seljuk period is that various arabesque designs, lines and geometric designs can be found in both the structure and the decoration of the building. The geometric designs of the decoration, initially based on simple brick layouts, but gradually structural geometry became more complex and more diverse. Despite the many devastation and political instability with the Mongol invasion, which led to the isolation of architectural artists, the decorations undergo the same simple process, with the difference that color tiles also become commonplace, and geometric designs appear more complicated than before. For example, during the pre-Ilkhanids period (Khwarazmshahian), the Gonabad mosque, which seems to have been based on the practical knowledge of geometry in its drawing and designing of brick designs, were on the basis of mathematical knowledge (Figure 34), or Bayazid Bastami Mosque in the Ilkhanids era where the sample of its linear nodes is shown below (Figures 35 and 36).

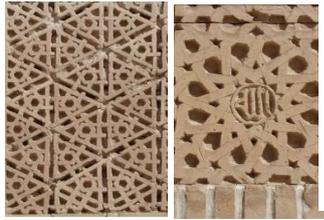


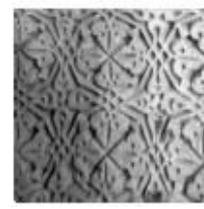
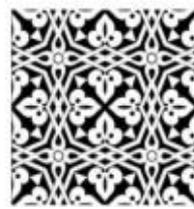
Figure 34. Geometric brick decoration of Gonabad



Figure 35. The linear node of six in the altar of the Bayazid Bastami Mosque



Figure 36. The linear node of eight and four leaves in the altar of the Bayazid Bastami Mosque



This complex process was considerably enhanced in geometric designs during the Ilkhanids period in the field of plaster; particularly, the Mongolian government, with the thoughtfulness of Iranian ministers, led to greater social political stability than before. As a consequence, pargetting replaces brickwork with the same arabesque spandrel, geometric and calligraphy designs. It should be noted that the prevalence of the pattern of geometric and arabesque designs in Islamic Iran was due to the theoretical foundations of Islam and their contradiction with iconography and drawing of organs. Therefore, geometry and mathematics become commonplace in Iranian architecture, and it is combined with art and the architect can create complex geometries in the absence of modern instruments and tools. He uses the geometric shape of the circle as the base of propagation of its various designs and effects, which symbolizes the creation of the multiplicity of the creators of God from unique God (Almighty God) in the universe. As a result, the geometric patterns of the 6-8-10 point and the more unusual 5-7-9-11 patterns are used in architecture (Figure 37 - 38)

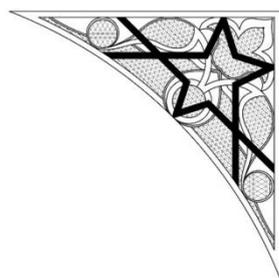


Figure 37. Linear pattern of the five-pointed knot in the altar of the tombstone of the Pirbakrān Isfahan



Figure 38. The second facade vault in the altar of the tombstone of the Pirbakran Isfahan

In ancient Persia, construction of astronomical structures and calculations was done using geometry. The Greeks evolved this knowledge and Euclid published all his knowledge of geometry in his first treatise, which was available to all in the late 2nd century AH. Geometry in the Muslim world was also of great importance because geometric shapes and structures were expressed in terms of cosmic and philosophical symbolic concepts and following the principles of geometry in decorations was the base of harmony which was a feature of Islamic art. The breadth and complexity of geometric

designs during the Ilkhanids period is also the result of the artistic architect's use of the knowledge of geometry, which was at the experts' disposal at the time. According to the author's extensive research on how this event took place, writing the book of the *Principles of Euclid*, which at the same time was translated from Greek to Arabic by Nasir al-Din, was the link between the art of architecture and geometry in this period. Chapter four of this book is devoted to teaching the drawing of peripheral environmental polygons, which marks the beginning of the application of geometric drawings in the field of architecture is a very good guide for students and experts. It should be noted that this book was written by Euclid in the year of 300 BC consisting of 15 chapters and was translated into Arabic during the Ilkhanids era by Nasir and sometime after into Persian by one of his students. Before Khajeh, of course, there were several descriptions of this book written by Jaberebne Hayan, Farabi, Abol Abbas-Fazl-Iben Hatam Tabrizi, and the most important note was by Avicenna in the mathematical section of the *Healing*. However, the reason for the use and prosperity of Khajeh's writing was the educational and practical application of the book as well as his position as the head of the educational institution of astronomy and mathematics, that is, the Mirage Observatory by Khajeh, during that era. As a result, the competent management ability of the Iranian philosopher during the ruling of the Mongols led the society to stability and peace in which the translation and compilation of various books, including the *Principles of Euclid*, provided the conditions for the growth and advancement of Iranian art and architecture, especially in the field of decorations. In fact, it can be said that the interaction of the architectural artists with the mathematicians of the time, which was the result of Nasir's prudence, created a turning point in the art of the Ilkhanids architecture, in particular the pattern of geometric decorations.

\* Source of figures: 10-11-18-19-22-21-22-23-24-25-26-27-30-31-32-33-35-36 (Kakhki & Taghavi Nejad, 2016).

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