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Cultural and Mathematical Meanings of Regular Octagons in Mesopotamia: Examining Islamic Art Designs

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Abstract

The most common regular polygon in Islamic art design is the octagon. Historical evidence of the use of an 8-star polygon and an 8-fold rosette dates back to Jemdet Nasr (3100-2900 B.C.) in Mesopotamia. Additionally, in ancient Egypt, octagons can be found in mathematical problem (Ahmose papyrus, Problem number 48), household goods (papyrus storage), architecture (granite columns) and decorations (palace decorations). The regular octagon which is a fundamentally important element of Islamic art design, is widely used as arithmetic objects in metric algebra along with other regular polygons in Mesopotamia. The 8-point star polygon has long been a symbol of the ancient Sumerian goddess Inanna and her East Semitic counterpart Ishtar. During the Neo-Assyrian period, the 8-fold rosette occasionally replaced the star as the symbol of Ishtar. In this paper, we discuss how octagonal design prevailed in the Islamic region since the late ninth century, and has existed in Mesopotamia from Jemdet Nasr to the end of third century B.C. We describe reasons why the geometric pattern of regular polygons, including regular octagons, developed in the Islamic world. Furthermore, we also discuss mathematical meanings of regular polygons.

Keywords: Islamic art design, Metric algebra, Regular octagon, Kharaqan towers, Inanna.

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Introduction

Regular octagons can be found in the works of Pierre Paulin (1927-2009), the ceiling of the Paris Opera House (19^{th} century), and the facade of the UMS building, Tashkent (Fig. 1). In Islamic art, these designs are called the 8-fold rosette, 8-star polygon, and 8-point star, respectively (see Fig. 5).

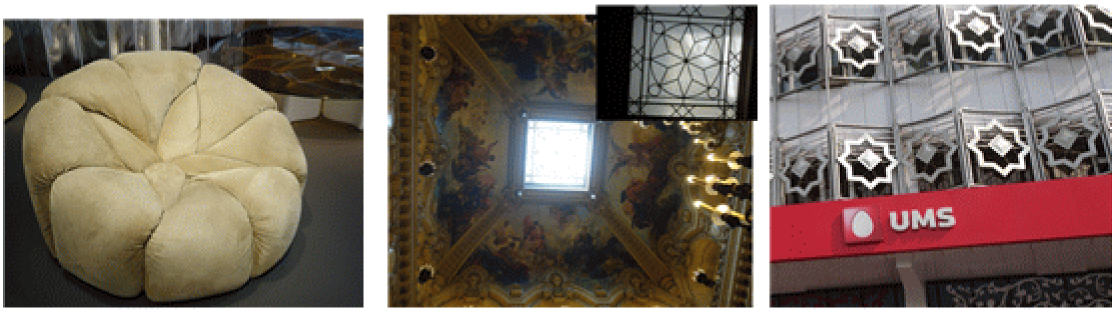


Figure 1. *(a) A Pierre Paulin's work (b) The Ceiling of the Opera House in Paris (c) UMS building (Photo: J. Park)*

Islamic art design can be classified into calligraphy, geometric patterns of polygon and arabesque. There are two types of calligraphy: kufic script (see Fig. 2) and cursive script.

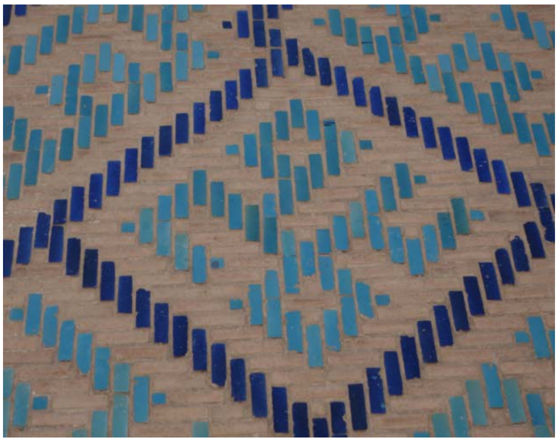


Figure 2. *Arabic Script of "Muhammad", the Gur Emir, Samarkand (Photo: J. Park)*

Geometric patterns of polygons can be found as abstracted elegant combinations. In Uzbekistan, examples of elegant combinations are prevalent. Regular 5-, 6-, 8-, and 16-gons are used in Ulugh Beg madrassah (1417-1420), and regular 4-, 5- and 6-gon are found in Gur-i Emir (1403-1404) (Fig. 3). Regular 5-, 6-, and 7-gons are used in the dome of the madrassah of Abdullah-khan in Bukhara (1588-1590) (Makovicky, 1986: 971). We can also find the elegant combinations of regular 5-, 6- and 10-gons in the Kukledash madrassah in Tashkent (mid-sixteenth century) (Fig. 4).

Figure 3. *(a) Ulugh Beg Madrassah (1417-1420) (b) Gur Emir (1403-1404) (Photo: J. Park)*

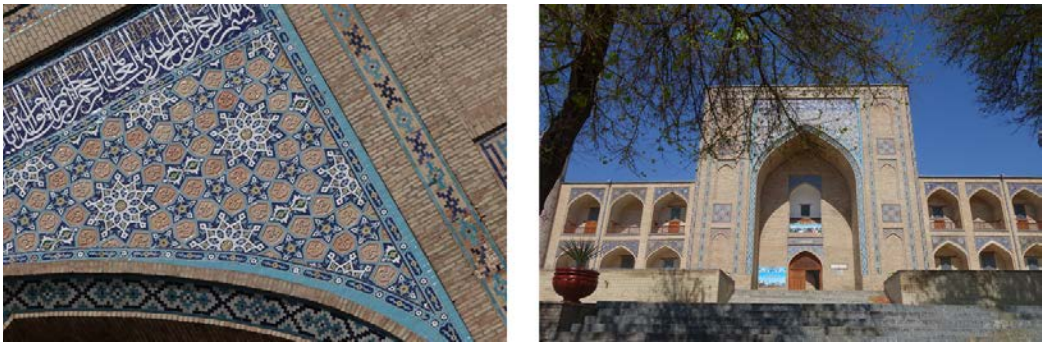


Figure 4. *Kukledash Madrassah (middle of sixteenth), Tashkent (Photo: J. Park)*

The history of Islamic geometrical ornaments is characterized from the beginning of the seventh century to the end of the ninth century, and at the end of the ninth century, geometric motifs were prevalent with Muslim architects and artists (Abdullahi and Embi, 2013: 244, 246). Geometry plays a central important role in Islamic art designs. The construction of star polygons, which are the most important elements in Islamic art design, is completely determined by circles. Critchlow (1999: 8) states that the circle is the primary cosmological symbol, one of wholeness and unity, and Abdullahi and Embi (2013: 245) claim that the circle is a symbol of religion that emphasizes One God and the roll of Mecca.

In this paper, we examined how the octagonal design, that prevailed in the Islamic region since the late ninth century, has existed in Mesopotamia, and why the geometric pattern of regular polygons including the regular octagon have developed in the Islamic world. Furthermore, we also discuss mathematical meanings of regular polygons.

The existence of 8-star polygons and 8-fold rosettes dates back to the days of Jemdet Nasr in Mesopotamia (Wolkstein & Kramer, 1983: 184, Friberg, 2007a: 166). Additionally, in ancient Egypt, the use of octagons including pseudo-octagons can be found in Ahmose papyrus (Problem number 48, late seventeenth century B.C.), household goods (papyrus storage, around 1400 B.C.), architecture (granite columns), and decorations (palace decorations).

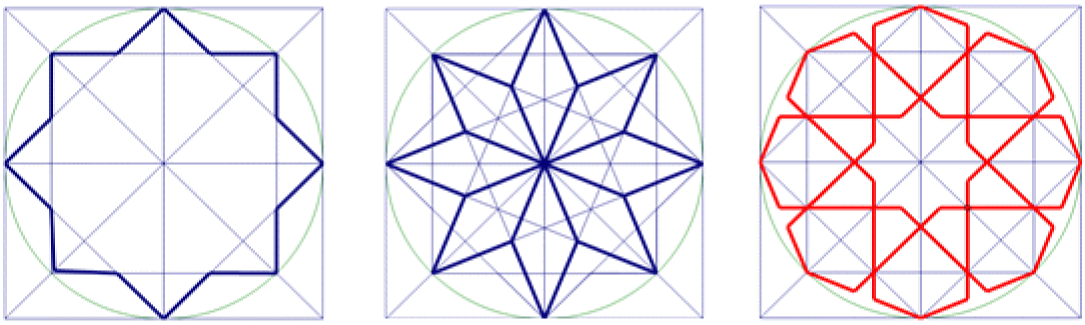


Figure 5. *(a) 8-point star (b) 8-star polygon (c) 8-fold rosette*

(Drawing: J. Park)

The regular polygon is a fundamentally important element in Islamic art design, and is commonly used as arithmetic objects in metric algebra in Mesopotamia. The 8-pont star or 8-fold rosette has long been a symbol of the goddess, Inanna/Ishtar, symbolized by Venus. Inanna's symbols appear in a stamp seal from the Gawre period (around 3300 B.C.) (Wolkstein & Kramer, 1983: 190). The regular octagon is believed to have developed in the Islamic world by integrating art, mathematics, and religion. This is consistent with Ettinghausen's assertion that the burgeoning Islamic civilization had to borrow from earlier traditions due to the lack of its own traditions (Lewis, 2002: 72). Many features, such as polygonal designs, which are regarded as typically Islamic go back to the time before Muhammad.

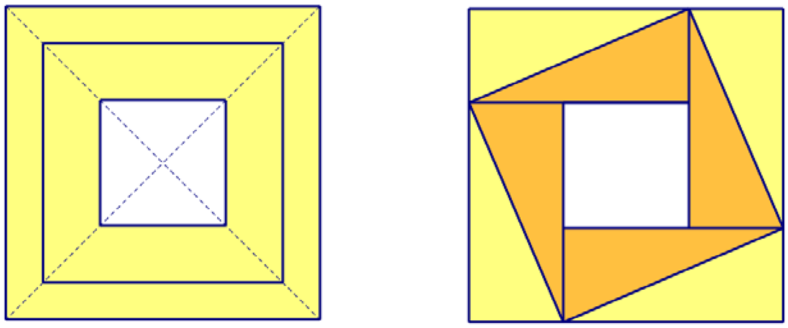
Regular polygons, Metric Algebra, and Point Symmetry

In Old Babylonian mathematics, metric algebra is an appropriate term for the special kind of mathematics combining geometry, metrology, and first and second degree equations. Metric algebra has been widespread from the time of the proto-Sumerians (end of the fourth millennium B.C.) to the Seleucid period (end of the first millennium B.C.) (Friberg, 2007a: vi). Regular 3-, 4-, 5-, 6-, 7-, and 8-gons are very important objects in Babylonian metric algebra. Old Babylonian mathematicians were adept at constructing regular polygons of appropriate size and formalized expressions such as:

$A_5 = 1;40 \cdot {s_5}^2, \;\; A_6 = 2:37,30 \cdot {s_6}^2, \;\; A_7 = 3\,;41 \cdot {s_7}^2,$

Where A_n denotes the area and s_n denotes the side of a regular n-gon (Friberg, 2007a: 161). In particular, the regular hexagon was used to find the circumference of a circle (Neugebauer, 1957: 47). A clay tablet states that the circumference of an hexagon is 0; 57, 36 times the circumference of the circumscribed circle. In this case, $\pi \approx 3\frac{1}{8}$ which is a better approximation of π than 3.

The Old Akkadian square band between the two given concentric squares is prominent in metric algebra (Friberg, 2007a: 77-8; Huber, 1955: 106). Based on the geometric interpretation of the solution algorithm for the metric algebra problems in VAT 8521 and Eshnunna text IM 67118, Friberg offers two methods, as displayed in Figure 6, of constructing a square halfway from the Old Akkadian square band (Friberg, 2007a: 78).



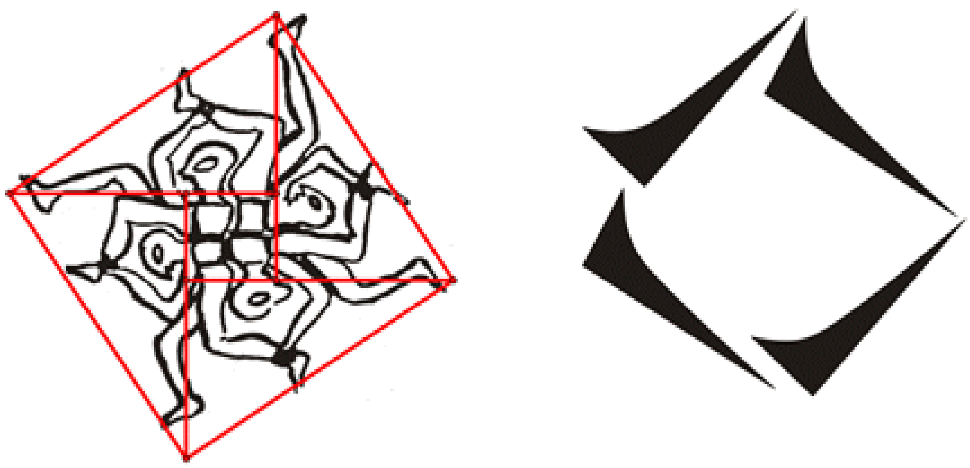
(a) (b)

Figure 6. *The Two Ways of Constructing a Square Halfway form the Old Akkadian Square Band (Drawing: J. Park)*

Figure 7. *A Way of Constructing Halfway from the Old Akkadian Square Band and Sidewalk Block, Tashkent (Drawing and Photo: J. Park)*

Babylonian astronomers used the equation associated with Figure 6(a) to compute the time at which Jupiter would have moved halfway along its ecliptic 60-day's path (Ossendrijver, 2016: 484). Moreover, Friberg proposes Figure 6(b) as a possible Old Babylonian proof of the Pythagorean theorem based on the basic idea of the solution procedure in IM 67118 (Friberg, 2007b: 206). Some researchers suggested that Old Babylonian mathematicians had a name for a "chain of right triangles" like the one formed by the orange right triangles displayed in Figure 6(b). Such a point-symmetry of right triangles resembles the cuneiform number sign $\check{s}\acute{a}\mathrm{r} = 60 \times 60$ in appearance (Fig. 8(b)) (Friberg, 2007b: 206).

The point-symmetric motif within a square is a popular pattern in Islamic art (Critchlow, 1999: 72). Also, this point-symmetric motif goes back to the proto-Sumerian Jemdet Nasr period (see Fig. 8(a)) (Friberg, 2007a: 167). In particular, we will use Figure 6(b) to examine a brickwork geometric pattern in the tomb towers of Kharraqan.



(a) (b)

Figure 8. *Point-Symmetric Figure and sign $\check{s}\acute{a}r$ —sign (Drawing: J. Park)*

Since the number 7 is not a Fermat's prime number, the regular heptagon can't be constructed using a compass and straightedge (Dummit and Foots, 2004: 602). Old Babylonians roughly constructed the regular heptagon in metric algebra. The part of clay tablet TMS 2(rev.) was lost (see Fig. 9, below). However, the height of the isosceles triangle was presumed to have been recorded (Friberg, 2007a: 162-163). The length of each side of the perimeter of the hexagon was 30. It can be presumed that the circumference of the circle was approximately $7 \cdot 30 = 3\ \ 30$, so the diameter was $1\ \ 10$, and the radius 35.

Figure 9. *TMS 2(rev.), Louvre Museum (Photo: J. Park)*

Now, let's take a closer look at the construction process in TMS 2(rev.) (Fig. 9). The construction in TMS 2(rev.) was, apparently, obtained as follows. First, Old Babylonians drew a circle with a radius of 35, and displayed a small hole in the clay at each corner using a line or a bar of size 30 along the circumference. The Old Babylonian erased the circle after connecting with line segments. There is a trace of a circle erased in the upper left corner of Figure

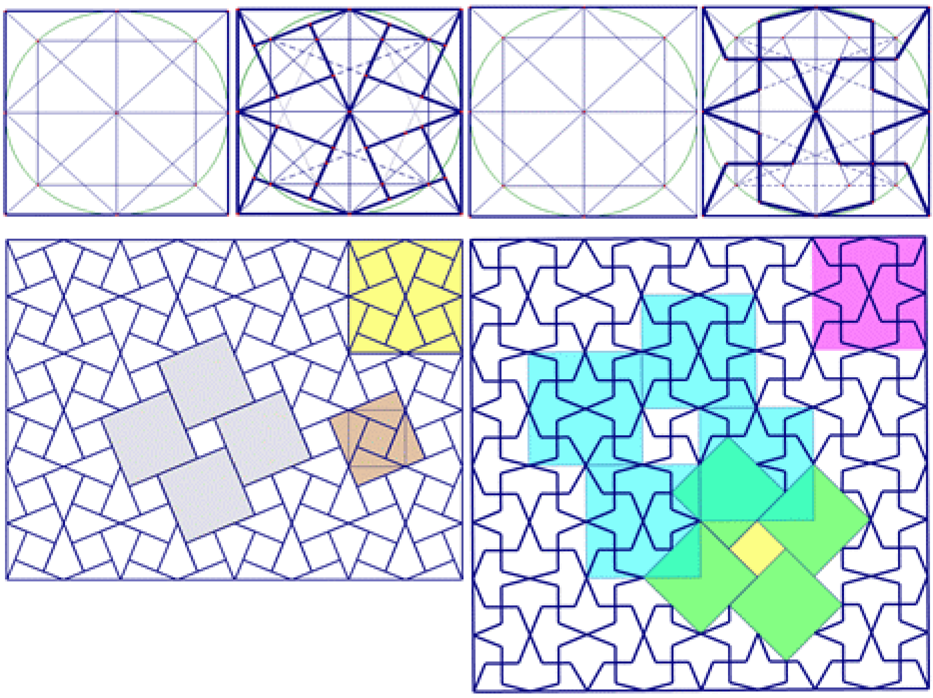
9. Under the assumption that the Old Babylonians were familiar with the octagonal form, Friberg suggests a detailed metric analysis of a regular octagon in terms of its sides (Friberg, 2007a: 155-159).

Regular Octagon and Its Islamic Art Design

Simple 6- and 8-gons as geometric patterns were used in the Mosque of Ibn-Tulun (Cairo, 879). Abstracted 6- and 8-gons can be observed in the tomb towers in Kharaqan (Iran, 1067), and in the Mosque of Al-Juyushi (Egypt, 1085) (Abdullahi and Embi, 2013: 245; Necipoğlu, 1995: 99).

El-Said and Parman identify two patterns composed of octagons (Fig. 10, below) and four patterns composed of hexagons, in the tomb towers of Kharaqan. In addition, they analyzed the geometric patterns used in Iran, Morocco, India, Turkey, Afghanistan, Yemen and Iraq from the 10th to 17th centuries (El-Said and Parman, 1976: Chapter II).

The abstract octagonal patterns of the tomb towers (Fig. 10) are classified as a 'ppm' in the wall paper group (Martin, 1982: 108).



(a) (b)

Figure 10. *Geometric Analysis of Octagonal Patterns from the Kharaqan Tomb Towers in Qazvin (Drawing: J. Park)*

Figure 10(a) and 10(b) include two-rectangle patterns which are classified as a 'p4' in the wall paper group. Such designs are common for sidewalks.

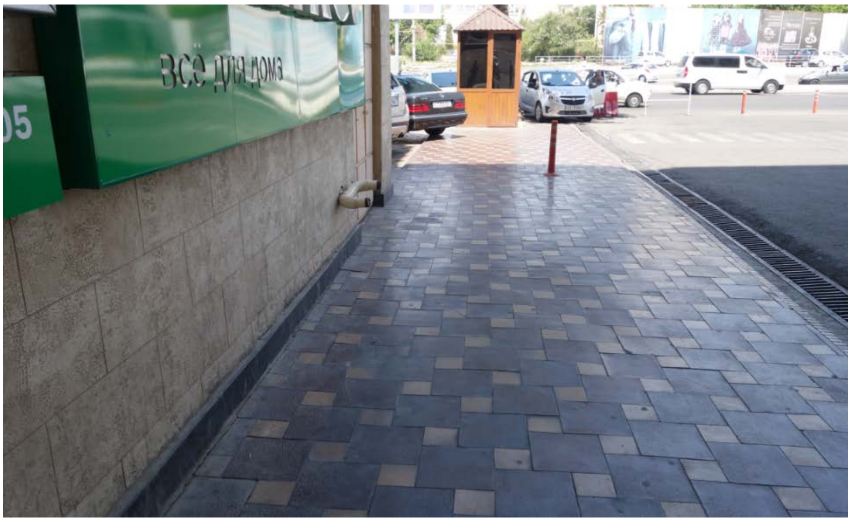


Figure 11. *Two-Rectangle Sidewalk Block, Tashkent (Photo: J. Park)*

In addition, Figure 10(a) contains a ring of four rhombuses (or whirling kites) (see Fig. 12) and octagonal decoration (see Fig. 13).

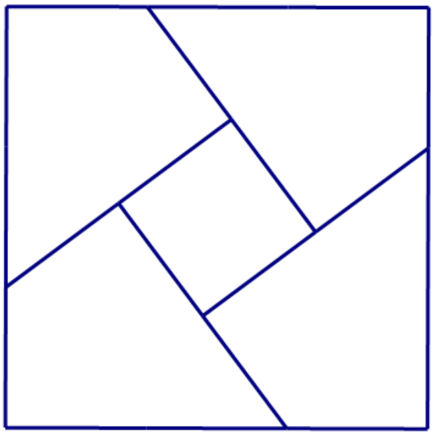


Figure 12. *A Ring of Four Rhombuses (Drawing: J. Park)*

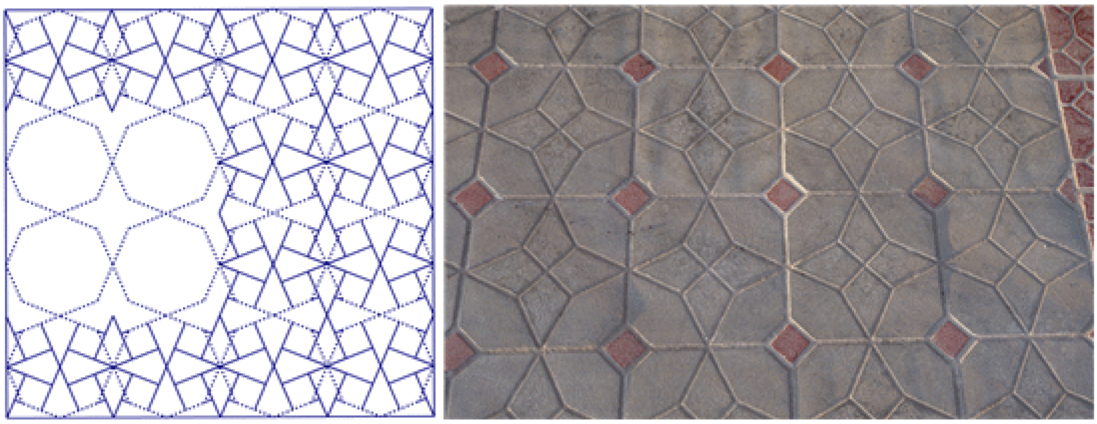


Figure 13. *Octagonal Interpretation of Brickwork Pattern from the Kharraqan Tomb Towers and its Sidewalk Block in Tashkent (Drawing and Photo: J. Park)*

Friberg (2007b: 206) proposes a geometric interpretation of IM 87118, which concerns determining the length of the sides of a rectangle when the area and diagonal are given, as in figure 6(b). Also, he updated the Høyrup and Damerow's list in which the diagonal rule is used either directly or indirectly (Friberg, 2007b: 450).

We suggest the possibility that the ring of four rhombuses design (Fig. 12) derived from the Old Babylonian square halfway (Figure 14(b), below), is related to the proof of diagonal rule, by inverting four right triangles located at each corner (painted parts in Fig. 14(b), below) (Park & Kim, 2017: 45). Therefore, we call the ring of four rhombuses (or kites) the altered Old Akkadian square band. This name reflects the mathematics (VAT 8521) of the Old Akkadian and the Old Babylonian mathematics (IM 87118).

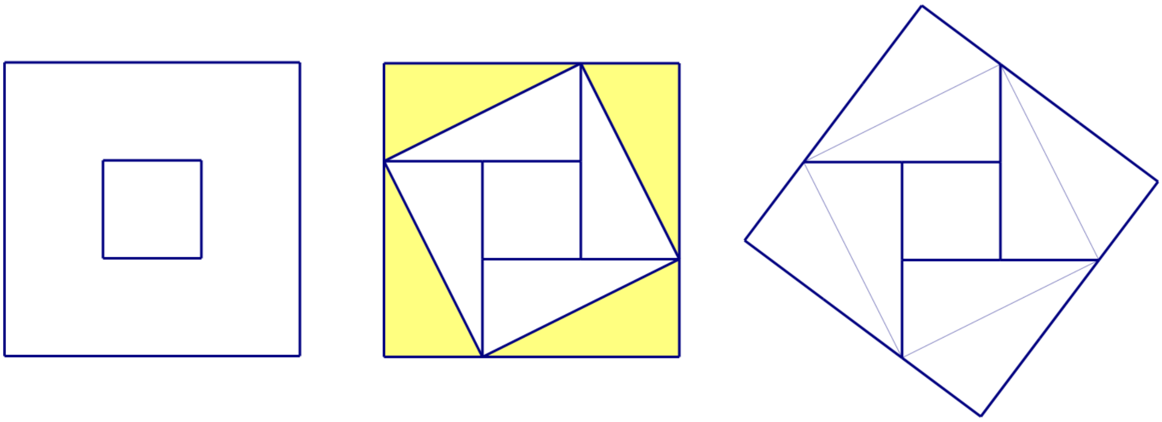


Figure 14. *(a) Old Akkadian Square Band, (b) A Square Halfway, and (c) Altered Square Band (Drawing: J. Park)*

Figure 15. *Wooden Door, A Ring of Four Rectangles at Khiva (Photo: J. Park)*

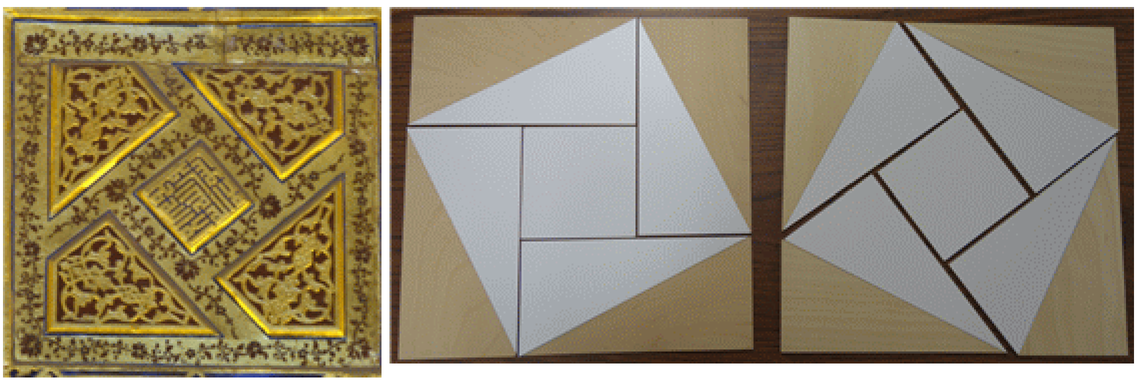


Figure 16. *Geometric Interpretation of the Ring of Four Rhombuses in the Tillya Kari Madrassah (1646-1660), Samarkand by the Old Akkadian Square Band (Photo: J. Park)*

Chorbachi (1989: 761-763) explains this spinning design in the Kharraqan brickwork pattern through a visual description and a sequential depiction of the underlying basic geometric structure present. On the other hand, Cromwell and Bertrami (2011: 86) claim that the ring of four rhombuses was influenced by the Chinese with respect to (Fig. 17). Zhao Shuang's commentary on the Chou Pei Suan Ching (China, 250 B.C.) contains an argument and diagrams in support of the Pythagorean theorem. However, the original diagrams are lost. Figure 17 displays what is believed to be "the Zhao's hypotenuse diagram" (Katz, 2009; 204-5).

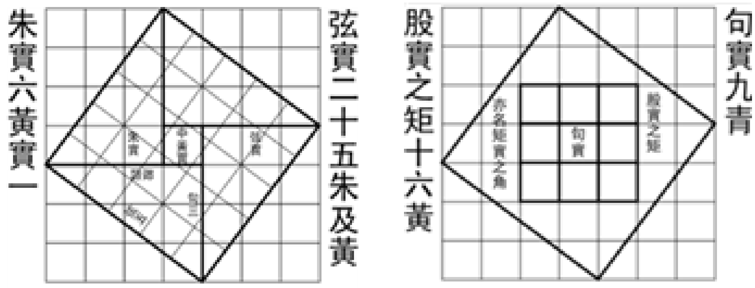


Figure 17. *An Estimating Picture of the Zhao's Hypotenuse Diagram ('Chinese Text Project')*

According to Friberg's doctrine based on the geometric interpretation of the IM 87118, it is reasonable to assume that the whirling kites were influenced by Old Babylonia rather than influenced by China.

Origins of Regular Octagons

In this section, we will examine 8-star polygons have existed in the periods of Jemdet Nasr, Akkad Dynasty, Old Babylonian, Neo-Assyrian, and Neo-Babylonian.

Figure 18. *A Ceramic Boal of Halaf Culture, Louvre Museum (Photo: J. Park)*

Figure 18 depicts reconstructed fragments of a painted ceramic bowl from the Halaf Culture (6000-5100 B.C.) found in Arpatchiyah, north of Mesopotamia (Louvre Museum Guide). It seems drawn reflecting the shape of a plant leaf, and so it is difficult to see the design as a geometric expression of an octagonal geometric shape.

The geometric recognition of the regular octagon with diagonals can be seen in seal imprints of clay excavated from layers below the famous royal cemetery in the ancient city of Ur (Friberg, 2007a: 166) (see Fig. 19 below). Legrain claims the production date as Jemdet Nasr (3100-2900 B.C. (Friberg, 2007a: 164). 8-star polygons can be found on a gaming board, known as the game of twenty squares, from the Early Dynastic Royal Cemetery of Ur

(Robson, 2008: 46). Also, there exist two 8-star polygons, the star symbol of the goddess Inanna, in a cylinder seal from Jemdet Nasr period (Wolkstein & Kramer, 1983: 184).

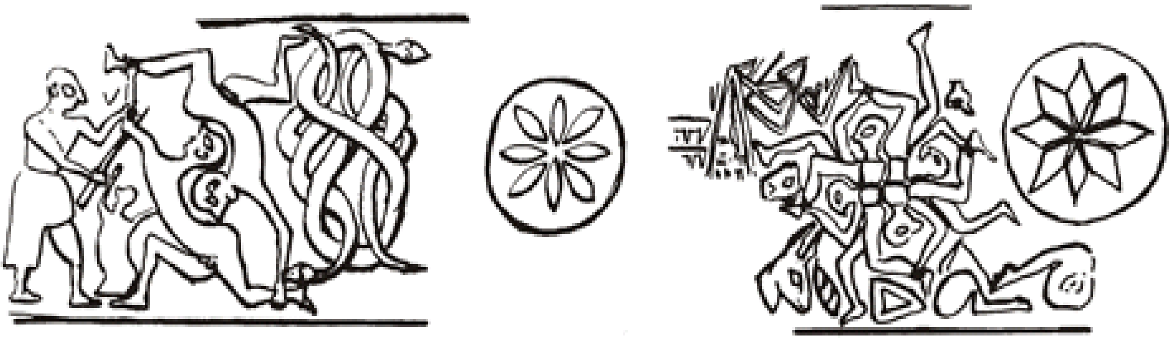


Figure 19. *(a) UE 3, 286 (b) UE 3, 393 (Drawing: J. Park)*

It is reasonable to consider Figure 19(a) as one of the first examples of an 8-fold rosette, considering each rhombic shape is elliptical. The elliptical shape may derive from a petal shape or a grain-field shape. Babylonians distinguished between the thin oval and thick oval, called grain-field and ox-eye (AEBO' in Fig. 20), respectively, as displayed in TMS 3 (Friberg, 2005: 134; 2007a: 323-4). The former was constructed using a square inscribed in a circle, and the latter was constructed using an equilateral triangle inscribed in a circle (see Fig. 20). These were also subjects of metric algebra.

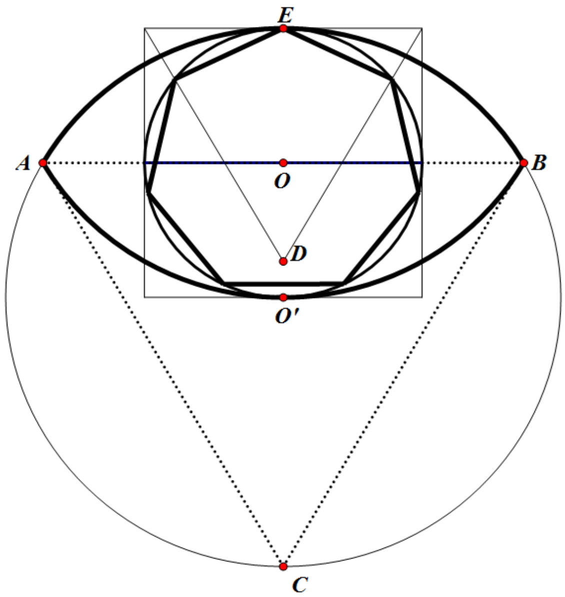


Figure 20. *A Construction of 7-gon with the Ox-Eye (Drawing: J. Park)*

Old Babylonian mathematicians determine the length of the minor axis (EO') and the major axis (AB), using $\sqrt{2} \approx \frac{17}{12}$ for the grain-field and $\sqrt{3} \approx \frac{7}{4}$ for the ox-eye (Friberg, 2005: 134, 135). The role of the ox-eye in geometry appears to be of great importance. Ox-eye determines the perpendicular bisector of a given segment. This method can also be found in ancient Egypt (Karpinski, 1915: 2).

On the other hand, the epic of Gilgamesh, widely known to be produced approximately 2000 B.C., includes the symbol of a bull symbolizing power in nine instances, and the 'Bull of heaven', the personification of the drought created by Anu for Ishtar, appears twice (Sandars, 1972: 87). In the Babylonian region, it is natural that a somewhat convex oval is called bull-eye.

In conclusion, Old Babylonian differentiated thin and thick ovals. Note that in the circumscribed rectangle of the circle center O (see Fig. 20), the point D makes an equilateral triangle. The intersections of the equilateral triangle and the small circle determine the two sides of the hexagon. Of course, it is not a regular hexagon, but it approximates 99.9% (Allen, 2013: 31).

Looking at the top of the monument of Naram-Sin (2254-2218 B.C.) (See Fig. 21(a)), a typical method of Mesopotamian art depicts war scenes using two solar discs designed as an 8-star polygon with striped rectangles between them. The solar disc, which was designed as an 8-star polygon, is also prominent at the end of the third century B.C. (see Fig. 21(b)). Figure 21(b) displays the distinctive style of the southern Mesopotamian region and was found at the Susa.

Figure 21. *(a) Top of the Monument of Naram-Sin (b) A Monument found in Susa, Louvre Museum (Photo: J. Park)*

In addition, Old Babylonians exhibit an octagram with its diagonals in IM 51979 (see Fig. 22, left) (Friberg, 2007a: 164). Concerning IM 51979, the 8-star polygon implies that the subgroup of Z_8 generated by 3 is Z_8 itself. The four lines passing through the center present the factor group $Z_8/4Z_8$. Since gcd(3, 8)=1, Old Babylonian mathematics draws all diagonals at once. In particular, since the number 8 is even, there is a diagonal line passing through the origin.

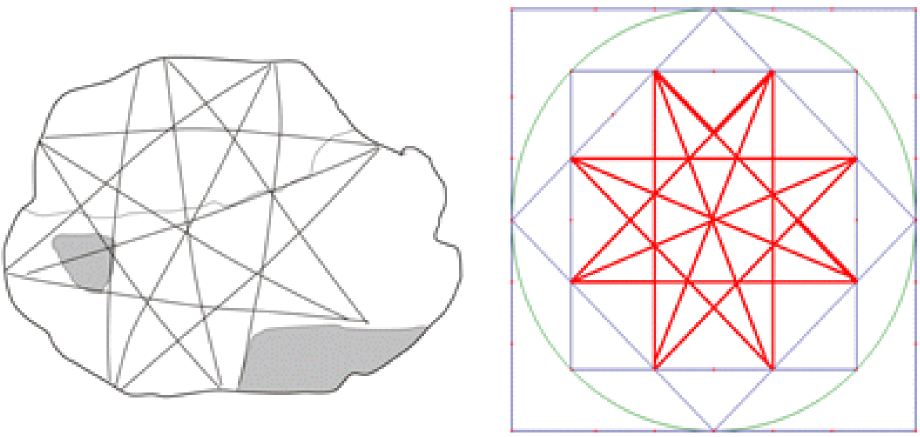


Figure 22. *Old Babylonian 8-Star Polygon, IM 51979 and Its Geometric Construction (Drawing: J. Park)*

An 8-point polygon was also found at the time of the Meli-Shipak (ruled 1186-1172 B.C.) during the Kassite Dynasty. In the Kassite Kudurru (Fig. 23), Meli-Shipak introduces her daughter to Shamash. The star hieroglyphics symbolize Venus (Ishtar), the moon (Sin) and the sun (Shamash) from left to right.

Figure 23. *Kudurru of King Meli-Shipak (ruled 1186-1172 B.C.), Louvre Museum (Photo: J. Park)*

During the Old Babylonian period, the 8-point star was enclosed within a disc (Black & Green, 2014: 170). In the frieze of archers (around 510 B.C.), we can find circles in Ishtar's 8-star polygons (see Fig. 24).

Figure 24. *Frieze of Archers, Darius I (around 510 B.C.), Louvre Museum (Photo: J. Park)*

For reference, let's look at examples of octagonal shapes used in ancient Egypt. The problem 48 of Ahmose papyrus concerns determining the area of a circle with a radius of 9. Text was written by Ahmose in the thirty-third year of the Apophis' reign (Chace, 1979: 84, Bernal, 2002: 328). There is a note on the back of the book (Chace, 1979: 119). It refers to "Birth of Set and Isis" in the eleventh year of an unknown pharaoh's reign. There is uncertainty, but Bernal (2002: 328) proposes 1628 B.C. as the eleventh year of Apophis base on the Thera eruption. According to his doctrine, we claim that Ahmose wrote the book in 1606 B.C. approximately.

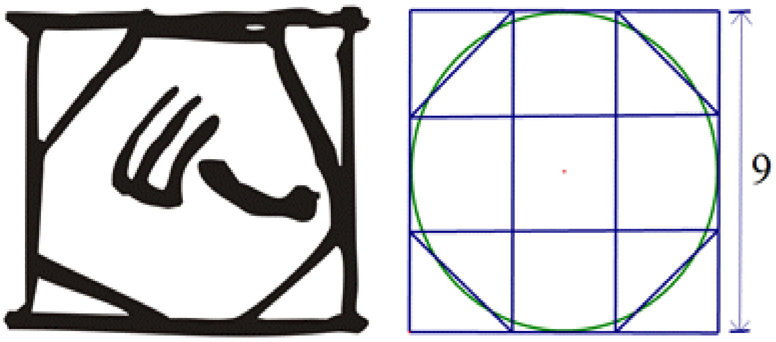


Figure 25. *Ahmose's drawing of Problem 48 and Vogel's Conjecture (Drawing: J. Park)*

According to the Vogel's claim, the area of pseudo-octagon is 63 (Fig. 25, right). It corresponds to a square whose side is $\sqrt{63}$, which is approximately $\sqrt{64} = 8$. Thus, the area of a circle formula $(8d/9)^2$ might have originated in which d is the diameter of circle (Gillings, 1975: 142-3). As a consequence, Old Babylonians found the area of a circle using a rectangle and octagon (cf. Diop, 1981: 243).

In the papyrus box (see Fig. 26) used around 1400 B.C. (Louvre Museum Guide), we can see the 4-, 8- and 16-star polygons, which seems complex but is based on octagonal construction. The lid can be rotated by fixing a point (Louvre Museum Guide). Looking at the design of star polygons in Figure 26, we can see that at least in the 15th century B.C., Egyptians used an 8-point polygon in their designs.

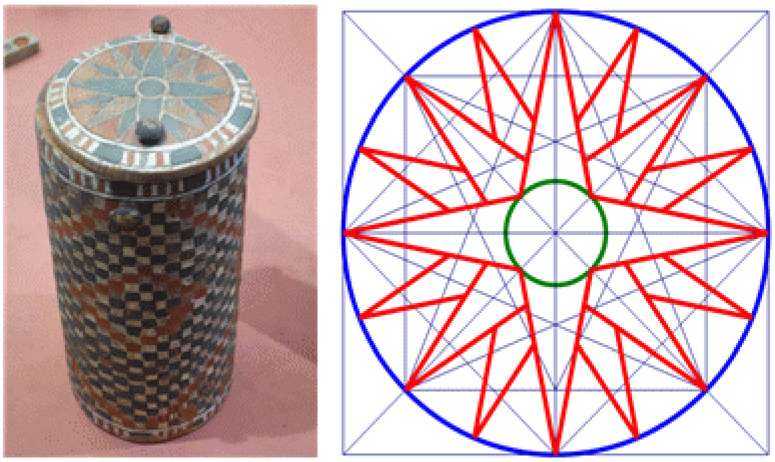


Figure 26. *Egyptian's Papyrus Box and Its Star Polygon Design, Louvre Museum (Photo & Drawing: J. Park)*

The cross section of a column of the temple at Philae is an 8-fold rosette (Diop, 1981: 298) and during the reign of Ramses III (1184-1153 B.C.), 8- and 10-fold rosettes were used as palace decorations. In particular, Babylonians found an area of the concave square (Katz, 2009: 16), which is used for adornment with the 8-fold rosette (Fig. 27, left).

Figure 27. *Palace Decorations and A Model of Cross Section of a Column, Louvre Museum (Photo: J. Park)*

Conclusion

The reason for the widespread use of octagons throughout the Muslim world is due to cultural interrelationships, natural environments, and the collaboration between mathematicians and artisans. We presented the reason in four ways as follows:

1. The symbol of pre-Islamic indigenous goddess was succeeded.

Inanna/Ishtar, the goddess of love and war, was the high divinity in Sumerian culture (Campbell, 2013: 79). In approximately 4000 B.C., the dominance of the Uruk was evident in two temple areas: the later shrine of Kullava and the early Eanna precinct, which later contained the shrine of Inanna. Other Sumerian cities had the custom of sending ritual offerings to the shrine of Inanna (Scarre and Fagan, 1997: 73-4). The most important document for defining the role and functions of Inanna during the fourth millennium is the Warka vase (Collins, 1994: 113). We can find two Inanna's 8-point stars on a cylinder seal from the Jemdet Nasr period (Wolkstein & Kramer, 1983: 184). During the Old Babylonian period, Venus was normally a symbol of Inanna/Ishtar, and the 8-point star was enclosed within a disk (Black and Green, 2014: 170). See the Inanna’s star displayed in Figure 28 below.

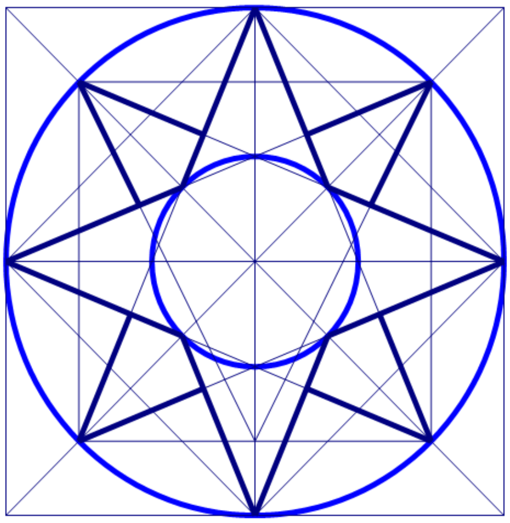


Figure 28. *Construction of The Inanna’s Star (Drawing: J. Park)*

During the Neo-Assyrian period the 8-fold rosette occasionally replaced the star as Inanna's symbol (Black and Green, 2014: 156-7). In Kudurru, an 8-point star is often placed beside the crescent moon, which is the symbol of Sin, god of the Moon.

2. Architects and artists used regular polygons to solve artistic conflicts.

At the end of the 9th century, the geometric theme was adopted by Muslim architects and artists. It is assumed that regular polygons played a role in satisfying their artistic conflicts. In public (or religious) art, geometric works have often been used in conjunction with plants, calligraphy, as well as portrait-free landscapes, and therefore, the regular polygon seems to have helped establish a decorative atmosphere in the otherwise monotonous surroundings and cityscapes of Islamic regions (Lewis, 2002: 96).

3. The mention of 8 in the Quran:

> (The Holy Quran, Chapter 69, verse 17) "And the angels will be on its sides, and eight will, that Day, bear the throne of thy Load above them"

This passage can be regarded as a psychological factor that easily accommodated the regular octagonal form of Islamic art.

Many scholars have commented on the Quran over many years, but there is no mention of the number 8. However, it can be interpreted as an important contribution to the acceptance of the octagonal form that has existed extensively in Mesopotamia and Egypt before Muhammad.

4. Much progress has been made in the Islamic art design with regular meetings of artists and mathematicians.

Mathematicians such as Abulwafa (10th century), Omar Khayyam (11th century), Al-Kashi (15th century), and Efendi (late 16th-early l7th century) had meetings with artists in Baghdad, Isfahan, Samarkand, and Istanbul, respectively (Özdural 2000; 171-172). For example, Omar Khayyam solved a cubic equation related to a special right triangle, in which the hypotenuse is equal to the sum of the short side and the perpendicular to the hypotenuse. Omar Khayyam supposed that $\overline{CD} = \overline{BC} + \overline{BE}, \overline{CE} = 10$ and $\overline{BE} = x$ (Fig. 29, left).

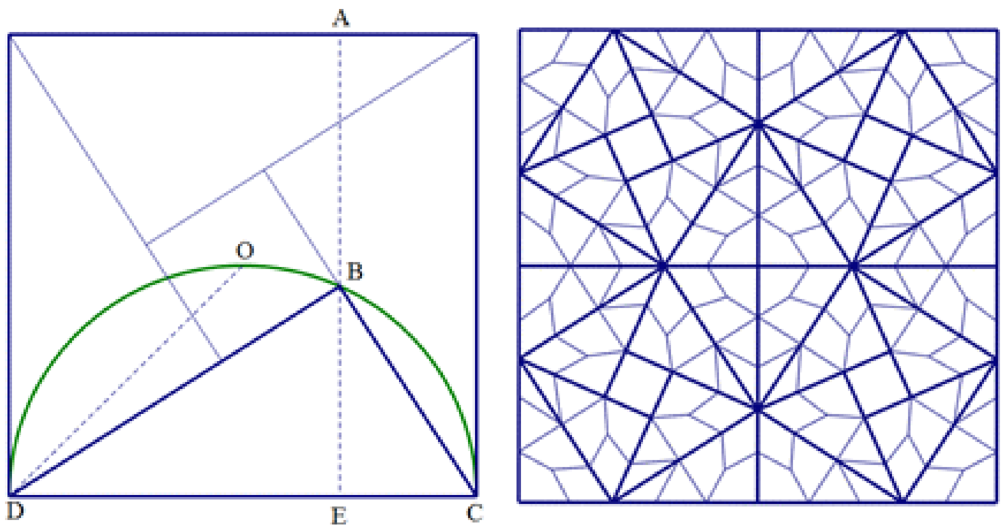


__Figure 29.__ Omar Khayyam's Triangle by the T-Ruler and Persian Manuscript 169, fol. 191r (Drawing: J. Park)

Therefore, he reduced the question raised at a meeting to find the solution of $\mathrm{x}^3 + 200x = 20x^2 + 2000$ by the intersection of a circle and a hyperbola (Özudural, 1995: 58, 59). As a terminology of Omar Khayyam, it is "a cube and roots are equal to squares and numbers" Also, he showed that if $\overline{AB} = \overline{BC}$ by the T-ruler, then $\overline{CD} = \overline{BC} + \overline{BE}$ (Özudural, 1995: 59, 63) (Fig. 29, left).

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